Modelling desert dune fields based on discrete dynamics

Hiroshi Momiji(1) *, Steven R Bishop(2),
Ricardo Carretero-González(2) † and Andrew Warren(1)

(1)Department of Geography, University College London,
26 Bedford Way, London, WC1H 0AP, United Kingdom
(2)Centre for Nonlinear Dynamics and its Applications,
University College London,
Gower Street, London WC1E 6BT, United Kingdom

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[*] Corresponding author: Tel: +44-(0)171-504-5500; Fax: +44-(0)171-504-4293; E-mail: hmomiji@geog.ucl.ac.uk

[†] Present address: Department of Mathematics & Statistics, Simon Fraser University, Burnaby, BC, V5A 1S6, CANADA

abstract

Two mathematical models to simulate desert dune fields based on discrete dynamics are proposed. Both models are developed by simplifying two mechanisms; sand transport induced by wind and gravitational shaping. Despite complex wind flow over desert dune fields some recurrent, spatial features occur. These models comprise our understanding of the effect of various mechanisms and succeeded in vividly demonstrating some important aspects of dune field. For model validation, comparison between simulated results and nature is necessary on different length scales [80 words].

Key words: desert dune field, modelling, discrete dynamics

1 Introduction

Mathematical modelling is becoming a common tool in many fields of study. Some use reconstruction techniques to develop a model from know data hence comprising our understanding of physical interactions, and others employ it to predict future states, for example, possible hazards. Restricting ourselves to a study in the geoscientific field only, the usefulness of mathematical modelling has been widely recognized through its applications to such topics as hill slope evolution, wind erosion estimation and wind ripple formation. Here we are interested in the morphology, dune growth, downwind migration and spatial patterns which form in a sand field, where we hope to clarify the physical features which play the most important role.

Dune fields show fascinating features which attract the research attention of scientists. For physicists the dynamic self-organized patterns, in which we can see, for example, regular spacing with defects embedded, may be very attractive. In terms of physics, a dune field is a typical non-linear open system (for an example introductory review from this view point see Werner 1999b). A field experiment shows that without continuous sand input a barchan dune, a type of dune explained in the next section, cannot maintain its shape (Cooke *et al.* 1992). Recently two modelling approaches, reviewed below, specifically consider such features.

Although it is well known among geomorphologists that vegetation cover and pre-existing topography play significant roles in aeolian land formation (see for example Cooke *et al.* 1992), we simplify the problem and restrict our interest to dunes in a perfectly dry and initially almost flat desert. These dunes are called free dunes. In such a dune field, any landform is a result of recurring interaction between wind flow over the dune field, topography, and gravity. Despite many excellent descriptive works (for example Kocurek *et al.* 1992), only a few modelling attempts have succeeded even in reproducing a single dune.

When considering the whole process of dune field formation scale is a big issue. A dune field extends for more than hundreds of kilometers, and takes more than hundreds of years to be formed. Unlike the study of microscale geomorphology represented by wind ripple, in the study of this large-scale geomorphology it is very difficult to investigate the physical processes through well defined experiments. The alternative way is to assume a set of plausible physical processes based on observations, create a model, and then compare simulated results to nature.

Difficulties arise in simulated results due to the strong non-linearity involved in the system, which includes wind flow over a dune field expressed by the Navier-Stokes equation, sand transport induced by wind and sudden deformation by gravity known as avalanching, leading to non smooth dynamics in the resulting differential equations. As we shall see later, some dune-characteristic features such as strong asymmetric profile need theory, which goes beyond linear analysis.

If we want to describe the evolution of such system starting from first principals we have to consider momentum conservation of the system comprising air and sand grains. However no such equation is yet available. Even if we can consider wind flow and sediment transport separately, it is extremely difficult to solve the Navier-Stokes equation, even in the case of an isolated single dune on a flat surface. In the case of a dune field comprising many dunes with different shapes, it is almost impossible to calculate the wind flow with same accuracy as in the case of single dune. Furthermore, we have to solve these equations under moving boundary conditions as dunes grow, in a three-dimensional space. Tracking dune evolution through the conventional analysis by calculating wind flow over a dune filed each time is therefore practically impossible. To simulate dune fields, we need more phenomenological models based not on detailed physical processes, but on simplified rules which, nevertheless correctly reflect observational facts.

Such models have been recently proposed by Nishimori and co-workers (Nishimori and Ouchi 1993, Nishimori *et al.* 1998) and by Werner (1995). both models are formulated with discrete dynamics. Though their mathematical formulations are not the same, they both assume that a dune field can be considered to be an accumulation of sand 'slabs', an imaginarily introduced sand block, piled in a two-dimensional lattice, and that the movement of these slabs can be used to simulate wind directional sand transport and deformational shaping by gravity. Both these models succeeded in reproducing some types of dunes, and explaining the relation between dune shape and its forming environment.

In this paper, firstly dune and dune field geomorphology and related aeolian processes are briefly reviewed. The modelling concepts are reviewed, where two dune field models developed by Nishimori and co-workers and by Werner are described and examined in detail. Then through authors' recent work, it is shown that introducing physically motivated modifications makes the model more realistic. Finally comments on remaining problems are made.

2 Aeolian processes and dune field geomorphology

2.1 Wind flow over a dune

Wind flow over a terrain can be described as boundary-layer flow, which is characterized by wind shear velocity, u_*. When the terrain can be approximated with an isolated low hill whose windward and leeward slopes are gentle enough, surrounded by a plane surface, the wind flow pattern is predicted by the Jackson-Hunt theory (Jackson and Hunt 1975). This theory's most important result is that the wind speedup over the windward slope and wind decline over the leeward slope, which come from the stream line convergence and divergence, respectively. Linear wind speedup against distance is observed by Lancaster (1996) and Mulligan (1988) for the windward slope of transverse dunes and barchan dunes, respectively. However applicability of this theory is limited. In the case of fully matured dunes with its slip face in the lee, wind separates from the dune at the dune crest and the reverse flow occurs in the lee, neither of which can be predicted by this theory.

2.2 Sand transport induced by wind and by gravity

There are four distinct types of sand motion induced by wind. All of them are triggered and maintained by wind shear stress working on the sand surface. The dominating sand transport motion for dune dynamics is ballistic sand grain motion called saltation whose typical flying distance is order of *cm*. Saltation-dominated sand transport can be described by Bagnold's semi-empirical sand transport formula, which non-linearly relates the sand flux, $q(x)$ to the wind shear velocity, $u_*(x)$:

$$q(x) = C(d/D)^{1/2}(\rho_a/g)(u_*(x))^3, \tag{1}$$

where C is a constant, d is sand grain diameter, D is reference diameter of 0.25 *mm*, ρ_a is air density, g is gravitational acceleration (Bagnold 1941). Together with the Navier-Stokes equation, this equation describes momentum conservation of the system. In the lee of the dune, avalanching, another sand transport induced by gravity, occurs when the slope exceeds a certain angle, typically taken as 33°. As a result of this catastrophic shaping, a slip face with a straight cross section is formed.

2.3 Sand conservation equation

Dune dynamics, which describes changes of dune shape and dune migration is governed by the sand conservation equation (often referred as sediment conservation equation):

$$\gamma\, \partial\eta/\partial t + \partial q/\partial x + \partial q/\partial y = 0, \tag{2}$$

where γ is the bulk density of sand, η is surface elevation and q is sand flux. This is equivalent to mass conservation equation. From this equation we know that erosion occurs when the sand flux, q diverges, and on the contrary, deposition occurs when it converges.

2.4 Dune migration

If dunes migrate downwind at a constant speed, c_d without changing their shapes, the dune migration speed, c_d can be written as

$$c_d = T_E\,(u_{*cr}, H)\, q_{cr}(u_{*cr})/\gamma\, H, \tag{3}$$

where T_E is the sand trap efficiency, the proportion of moving sand trapped in the dune slip face (Cooke *et al.* 1992), q_{cr} is the sand flux at the crest, H is the dune height (see for example Momiji and Warren 1999) and u_{*cr} is the wind shear velocity at the crest. The relation between dune migration speed, c_d and dune height, H is examined by Nishimori *et al.* (1998) in their modelling work reviewed below.

2.5 Classification of simple (elementary) dunes

Free dunes can be classified into three groups, depending on the alignments of their crest to the net sand transport; transverse, linear and star dunes. For transverse dunes, net sand transport is normal to the crests, where this group also includes barchan dunes. For linear dunes net sand transport is parallel to the crests, and includes so-called seif dunes. For star dunes, net sand transport is towards nodes. This group includes 'star' (large) and network (smaller) dunes. Each type of dune is described in detail below, and is shown in figure 1.

Transverse dune:	dune ridges with crests transverse to the dominant wind, which migrate, for the most part, in the direction of the dominant wind. Transverse dunes, which appear in infinitely deep sand field under unimodal wind regime as seen below, are thought to be the simplest dunes in view of their shapes and configuration (figure 1(a)).
Barchan dune:	a free transverse dune with a crescentic plan-shape in which the crescent opens downwind (figure 1(b)).
Seif dune:	a sinuous, sharp-crested, linear dune (figure 1(c)).
Star dune:	a large pyramidal or dome-like dune with arms (figure 1(d)).
Network dune:	result of the overlap of a number of transverse dune systems, each aligned to a different wind in a complex annual regime (figure 1(e))

2.6 Forming environment of each type of dune

Dune types are primarily determined by the wind regime. Fryberger (1979) categorized annual wind patterns into three main regimes; 'unimodal', 'bimodal' and 'complex', with four subregimes. In 'unimodal' regimes winds blow from almost the same direction throughout the year. In 'bimodal' regimes the wind distribution comprises two modes. In 'complex' regimes the wind distribution comprises either more than two modes or no distinct mode.

The second important factor is thought to be the amount of sand available for dune building. Wasson and Hyde (1983) developed a diagram of dune types with respect both to wind variability and to sand availability. Later this diagram was refined by Livingstone and Warren (1996), and a variation is included in figure 2. These diagrams are on the whole accepted by many researchers.

Another type of relation between bedform alignment and directionally varying flows was proposed by Rubin and Hunter (1987). From a theoretical consideration they claimed that bedforms are aligned such that the gross normal sand transport throughout the cycle of wind directional variation is maximum. This relation was supported with field observations of aeolian dunes by Lancaster (1991).

One of the main purposes of dune field modelling is to examine these relations between dune form and forming environment, and results are shown in the next section.

3 Dune and dune field modelling

There have been many attempts to model dune evolution and migration. They can mainly be divided into two types. The first type, which are based on concrete physical processes, aims to reproduce an isolated dune on a flat surface mostly based on a computational fluid dynamics approach. An exception is Stam's analytical work (Stam 1996, 1997). The other type, which are based on more abstract mathematical rules, aims to model the dune field, where many dunes grow, vanish, migrate and interact with each other. These are modelled based on discrete dynamics.

3.1 Modelling an isolated dune on a flat surface

A single dune and dune fields are thought to evolve through the following recurring interaction between wind flow and dune form, together with avalanching.

0 (4). current landform (dune)

⇓ (Navier-Stokes equation)

1. wind flow

⇓ via shear stress (sand transport equation (1))

2. sand transport

⇓ via erosion/deposition due to spatial variation of sand flux

⇓ (sand conservation equation (2))

3. modified landform (dune)

⇓ via avalanching (if the slope is too steep)

4. shaped landform (dune)

Dune evolution can be simulated by iteratively solving these four steps. This type of models has been carried out by Howard and Walmsley (1985), Wippermann and Gross (1985), Zeman and Jensen (1988), Weng *et al.* (1991). Simulations usually start from an simple, gentle, initial topography such as a conical sand pile. For wind flow calculation, Navier-stokes equation is usually replaced by modified Jackson-Hunt theories to reduce computation time. These have been intensively revised. For the sand transport equation, Lettau-and-Lettau's formula, one of the modified

Bagnold's formulae known that best fits with experiments, are often used (Lettau and Lettau 1978, Greeley and Iversen 1985). Though the question of equilibrium shape was not solved by Wippermann and Gross, they succeeded in reproducing a barchan dune with a good geometrical agreement to natural dunes. Currently the most established model is one developed by van Dijk *et al*(1999). Though this is a two-dimensional model, it demonstrates the growth of the asymmetric two-dimensional dune with a slip face in the lee from an initially symmetric gentle hill. The resulting dune reaches an equilibrium dune height.

Stam (1996) developed a two dimensional analytical model corresponding to linear stability theory. Since this is an analytical model it provides clearer insights about the dune migration and growth mechanisms, and makes long-term predictions possible. The model shows that combining the Jackson-Hunt theory and a linialized Bagnold theory can explain dune's growth from a plane bed with a small undulation. It also shows that dunes migrate at a constant speed. There are three drawbacks in his model. Firstly, only sinusoidal migrating patterns can be analyzed. Secondly, due to the linialization, dunes show exponential growth. And lastly, the model shows that all wavelength components are unstable, and that the higher the frequency, the higher the component grows. Consequently the shorter-wavelength components dominate, so that the dune profile cannot maintain its profile. Later Stam developed a numerical model incorporating avalanching, which is expected to suppress short-wavelength components, into his analytical model (Stam 1997).

However the resulting dunes still do not resemble natural dunes. This implies that non-linear features are not negligible.

3.2 Modelling dune field dynamics

Study with this type of simulation models has the merit of dimensionality. Some models, as seen above, discuss two-dimensional dunes transverse to the wind, because dealing with three-dimensional features is mathematically difficult. On the other hand, developing a computer simulation model based on discrete dynamics in three dimensions is not quite difficult.

Two models have been proposed, one by Nishimori and co-workers (1993, 1998), and another by Werner (1995). These models are mathematically different in formulation, but both are based on two-dimensional discrete dynamics. They attempt to simulate dune field dynamics on a two-dimensional lattice (figure 3(a)), where dunes emerge from a surface which is almost flat but has small random undulations, and grow. Summary of both models is in table 1.

3.3 Nishimori's model

Nishimori and Ouchi (1993) developed a dune-field model based on two-dimensional Coupled Map Lattice, CML, which is a mathematical system of spatially connected local maps distributed on the each lattice site (Kaneko 1991). A CML deals with a continuous variable on each lattice site, and here this was taken as the surface elevation, height, η. Periodic boundary conditions are applied both in $i(x)$ and in $j(y)$ directions (figure 3(b)). The model comprises wind directional sand transport and diffusion, which is the deformational shaping by gravity introduced instead of avalanching. Sand transport is set such that the slab is transported farther if eroded from the windward surface, and less for if from the lee surface due to the derivative, $\nabla_i \eta_n$ according to:

$$\begin{aligned} L &= L_0 - b \tanh(\nabla_i \eta_n(i,j)) \\ \delta\eta &= \delta\eta_0 + b' \tanh(\nabla_i \eta_n(i,j), \end{aligned} \tag{4}$$

where $\eta_n(i,j)$ is surface height at (i,j) at time, n, L and $\delta\eta$ are transport length and height of the slab, respectively, and L_0, b, η_0 and b' are constants. It should be noted that this slab transport does not represents saltating sand grain motion, rather does the bulk sand transport over a long time period, and that the transport length, L here is much longer than saltation length. Erosion and deposition in conjunction with sand transport is expressed as

$$\begin{aligned} \eta_{n'} &= \eta_n(i,j) - \delta\eta \\ \eta_{n'}(i + L(\eta_n(i,j)), j) &= \eta_n(i + L(\eta_n(i,j)), j) + \delta\eta, \end{aligned} \tag{5}$$

where n' is the intermediate time step between n and $n+1$. The diffusion process is formulated as

$$\eta_{n+1} = \eta_{n'}(i,j) + D[1/6 \sum_{NN} \eta_{n'}(i,j) + 1/12 \sum_{NNN} \eta_{n'}(i,j) - \eta_{n'}(i,j)] \tag{6}$$

where NN and NNN represent the nearest and the second nearest neighbouring sites, respectively. By repeating these processes (4) - (6) dune field evolution is simulated. Hence the evolution is deterministic. In terms of CML, local dynamics, mapping represents erosion, and the spatial connections among lattice sites represent wind directional sand transport and subsequent deposition, plus diffusion.

Nishimori and Ouchi succeeded in demonstrating the formation of a barchan dune field. Though a barchan dune has a strong three-dimensional feature, in this model for directions not parallel to the dominant wind, only diffusion is taken into account. Diffusion which primarily represents leveling process by gravity seems to play a significant role in the formation of three-dimensional pattern. Furthermore through the linear stability analysis in a two-dimensional space, they show that a dune field pattern emerges as a result of competition between gravitational deformation in the notation of diffusion and sand convergence following the slab transport rules in equation 4. The analysis also shows that many bands of wavelength component destabilize simultaneously. Nishimori and Ouchi propose that this is the origin of rich variety of dune field morphology. In nature there exist dunes with smaller dunes superimposed, often referred as compound dunes. However the origin of this type of dune is controversial. Through the analysis of airborne photographs, one of the authors (A.W.) is confident that the different size of dunes did not formed simultaneously, but they reflect their different formation periods (Warren and Allison 1998). The major problem of this model is that diffusion and directional sand transport occur at the same time scale, though in nature avalanching occurs at faster time scales than bulk sand transport.

Later Nishimori *et al.* (1998) extended this idea to simulate various types of dunes under different environments. Two major revisions were made. They refined the wind directional sand transport rule considering the wind flow calculation over a two-dimensional dune as:

$$\begin{aligned} L &= \alpha\,(\tanh(\nabla_i \eta_n(i,j)) + 1) \\ \delta\eta &= \beta\,(\tanh(\nabla_i \eta_n(i,j)) + 1 + \epsilon), \end{aligned}$$

where α, β and ϵ are positive constants. Also they introduced wind directional change by periodically shuffling three sand transport directions perpendicular to each other, resulting in three types of wind regimes; uni-, bi- and tri-directional. Together with these revisions, they investigated the effects of sand availability. They succeeded in demonstrating the emergence of barchan, transverse, linear and star dunes corresponding to different imposed wind conditions and sand availability (figures 5 and 6). The simulated transverse dunes and linear dunes have terminations (defects, Y-junctions), which are also observed in nature. They also showed relations between dune shapes, wind directional variability, and sand availability, which are roughly confirmed by observations (Wasson and Hyde 1983, Livingstone and Warren 1996, figure 5.22 on p80). A slight difference between simulated results and Livingstone and Warren's diagram is in the case of a bimodal wind regime. In the simulations, linear dunes and transverse dunes emerge corresponding to small and

large sand availability, respectively, though in the diagram the relation is reversed. They not only demonstrated dune and dune field morphology, but also some aspects of their dynamics; smaller barchans move faster than bigger ones:

$$c_d \sim H^{-1.25},$$

dune-dune interactions such as coalescence, smaller dunes running up and down larger dunes, as in some observed cases.

3.4 Werner's Model

Werner (1995) developed a slightly different dune field model. Unlike Nishimori's model, the dynamics are stochastic, and the dune field consists of three-dimensionally discrete sand slabs (figure 3(a)). Consequently the height, η is discrete and proportional to the number of sand slabs piled on each lattice site, $h(i,j)$:

$$\eta(i,j) = h(i,j)\,/\,3,$$

where 3 is slab aspect ratio. Periodic boundary conditions are adopted (figure 3(b)). An erosion site is randomly chosen, and the eroded slab is transported downwind a specific number of lattice sites. Based on Bagnold's observation, a depositional probability difference was introduced between sites with at least one slab and sites without a slab. In view of modelling scheme, this is a significant difference from Nishimori's model, since a CML does not allow a 'mapping' depending on the state of mapped site, here if a slab is present or not. Furthermore, a shadow zone of

15°, in which the deposition probability was unity regardless of if there is a slab or not, was introduced (figure 3(c)), which corresponded to the reattachment point downwind which was six times as long as the dune height from the dune (Frank and Kocurek 1996). Together with this wind-directional transport, avalanching occurs on the steepest descent if the gradient between two adjacent lattice sites exceeds 30° (figure 3(d)). A change in the wind direction was simulated by periodically altering the sand slab transport directions. This rule seems to more correctly reflect nature than that in Nishimori's 1998 model, since in a real dune field the wind blows in the same direction in each season every year.

Werner succeeded in demonstrating barchan, transverse, linear and star-like dunes, changing wind directional variability, and sand availability (figure 7), though for some simulations such as the one for transverse dunes, the presumed sand depth was not thought to be sufficient (Wasson and Hyde 1983, Livingstone and Warren 1996, figure 5.22 on p80). Like those demonstrated by Nishimori *et al.*, the simulated transverse dunes and linear dunes had terminations, whose motions may significantly contribute to the dune trend against the wind (Werner 1997, 1999a). Werner also demonstrated some aspects of dune and dune field dynamics; dune-dune interactions such as coalescence; small barchans being 'calved' from the horns of larger barchans. He also showed that dune shape was insensitive to wind direction deviation, and that, however, widening the angle between two winds form different directions, results in dunes abruptly changing from transverse dunes to linear dunes. In addition under the condition in which linear dunes emerge, initially introduced transverse dunes and/or many terminations cannot survive and result in linear dunes with less than ten terminations. From this result, together with dune fields illustrating distinct patterns such as barchan dunes and linear dunes, Werner thinks that a dune field can be described with only a few number of variables, and that patterns such as barchan dunes are attractors of that space. Examining the resulting patterns in detail, even qualitatively, some problems can be found. For example, the cross-sectional views of simulated barchans and transverse dunes are much more symmetric than those of real dunes.

Neither paper quantitatively discussed the comparison between simulated patterns and real dune field patterns. This is thought to be because of the difficulty of scaling the dynamics in the model to the processes in nature.

It is also worth noting that de Castro (1995) developed a model to simulate the coastal dune field comprising alternating sequences of transverse dunes and vegetation area. The model basically resembles Werner's. However, some simulation rules adopted in de Castro's paper do not seem to be applicable to desert dune fields. Still it is interesting that the model deals with the open dune field with a constant sand supply from the sea, without setting a periodic boundary condition.

4 Towards an improved physical model

As discussed above both Nishimori's model and Werner's model were developed based on the fact that the bulk of sand grains are transported in the same direction as the wind flows, and on the fact that dunes deform by gravity, more specifically by avalanching. These can be considered to be the minimal model describing a dune field. Incorporating plausible physical mechanisms into these minimal models one by one, and comparing results with observations, it is thought to be possible to clarify physical processes involved in a dune field system, and to develop a more general model which resembles natures processes.

As the Jackson-Hunt theory predicts, the most prominent characteristic when the terrain intrudes into the boundary-layer wind flow is the wind speedup over the windward slope (Jackson and Hunt 1975). In their paper, Momiji and Warren (1999) emphasize this wind speedup over a dune and sand trap efficiency T_E, which is also related to wind speedup, as important factors when discussing equilibrium migration of transverse dunes. Momiji *et al.* attempted to incorporate this wind speedup effect into Werner's model which is thought to be more close to nature since it accounts for the time scale difference between bulk sand transport and avalanching, which occurs on a much quicker time scale.

From kinematic considerations, the wind speedup over a dune can be formulated by adding a term to the original transport length, which is proportional to the surface height, η:

$$L(i,j) = L_0 + \delta L_c\,(h(i,j) - h_{avg}), \tag{7}$$

where L_0 and δL_c are constants, (i,j) is the erosion/ongoing 'bounce' site and h_{avg} is the average number of slabs, taken in Momiji *et al.* as 100. Equation (7) can be interpreted as the linear wind speedup observed by Lancaster *et al.* (1996) and Mulligan (1988). Prior to the simulation, the definition of shadow zone is revised by introducing no erosion there.

Simulations show that introducing wind speedup results in: a) lower dunes; b) dunes migrating more quickly; and c) shallower dune windward slopes, producing asymmetric dunes, which closely resemble those in nature. Furthermore, simulations introducing a non-linear wind speedup term into equation (7):

$$L(i,j) = L_0 + \delta L_c\,(h(i,j) - h_{avg}) + \delta L_{c2}\,(h(i,j) - h_{avg})^2,$$

where δL_{c2} is a constant, shows the possibility that the dunes reach the equilibrium height.

Simulated dune fields yield flat surfaces between each neighbouring dunes. However some transverse dune systems observed in nature do not have distinct interdune flat surfaces. They discuss their origin and conclude that they are related to the shadow zone introduced in the model. To describe these features more precise dynamics in the lee of the dune must be incorporated.

5 Remaining problems

In computer simulations, dune-like patterns can be obtained with a variety of rules indicating that models which achieve qualitative agreement are not unique. Hence it is very important to examine if the simulation rules correctly reflect the real dynamics in nature. There has been no quantitative assessments yet either on Nishimori's model or on Werner's model. Through the comparison to nature, for example, the spatial scales of both models are expected to be clarified.

Another fundamental question is whether a dune field should be described by deterministic dynamics or by stochastic dynamics. We have to address if the stochastic rules employed in Werner's model, which appears quite successful, are essential in view of physics, or just a technical treatment.

As can be seen for barchan dunes and star dunes some features in a dune field are strongly three dimensional and simulations must be carried out on a two-dimensional lattice. This makes long-time predictions very difficult due to the lengthy calculations. Model analysis not relying on computers are thought to be becoming more important.

6 Conclusion

Two mathematical models to simulate desert dune fields based on discrete dynamics, originally proposed by Nishimori and co-workers and by Werner, have been further developed by simplifying two mechanisms; the sand transport induced by wind and gravitational shaping. Simulations vividly demonstrate some aspects of dune field consistent with field observations. Considering the difficulties in calculating wind flow over a complex desert dune field, these simple and phenomenological models are thought to be very promising minimal models. It is shown that by introducing physics-based modifications into the original works, for example by introducing wind speedup effect into Werner's model (Momiji *et al.*), models more resembling nature are available. For model validation, comparison between simulated results and nature is necessary.

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Table 1

item	Nishimori's model	Werner's model
model type	Coupled Map Lattice (CML)	conventional discrete lattice model
dynamics	deterministic	stochastic
boundary conditions	periodic	
surface height	continuous	discrete
sand transport	adjusted to wind profile over a dune	uniform
shadow zone	not introduced explicitly	introduced
deposition probability	unity all over the field	probability at a site with at least a slab is larger than that at a site without any
gravitational shaping	diffusion	avalanching
wind direction change	periodic shuffling	periodically sequential alternation

Figure captions

Figure 1 Photographs of typical types of dune; a) barchan dunes, b) transverse dunes, c) linear dunes, d) star dunes and e) network dunes.

Figure 2 Dune type diagram with regard to sand availability and wind directional variability derived by Livingstone and Warren (1996).

Figure 3 Model and algorithms; a) wind-directional sand transport, b) periodic boundary condition, c) avalanching and d) shadow zone.

Figure 4 Simulated barchan dune field by Nishimori's model.

Figure 5 Various types of simulated dunes with revised Nishimori's model; a) transverse dunes, b) barchan dunes, c) network dunes, d) seif dunes and e) star dunes.

Figure 6 Reproduced diagram between dune type and forming environment with revised Nishimori's model. The vertical and horizontal axes represent sand availability and wind directional variability, respectively.

Figure 7 Various types of simulated dunes with Werner's model; a) barchan dunes, b) transverse dunes, c) linear dunes and d) star dunes.

Figure 8 2-dim transverse dune pattern with wind speedup.