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DIFFERENT QUANTUM FIELD CONSTRUCTIONS IN THE $(1/2,0)\oplus(0,1/2)$ REPRESENTATION

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We present another concrete realization of a quantum field theory, envisaged many years ago by Bargmann, Wightman and Wigner. Considering the special case of the $(1/2,0)\oplus(0,1/2)$ field and developing the Majorana–McLennan–Case–Ahluwalia construction for neutrino, we show that fermion and its antifermion can have the same intrinsic parity. The construction can be applied to explain the present situation in neutrino physics.

Though it has been three decades since the proposal of the Glashow–Weinberg–Salam model, we are still far from understanding many of its essential theoretical ingredients; first of all, fundamental origins of "parity violation" effect, the Kobayashi–Maskawa mixing and Higgs phenomenon. Experimental neutrino physics and astrophysics provided us by new puzzles, that until now have not found adequate explanation. For instance, recently Prof. Bilenky[1] pointed out that following the analysis of the LSND neutrino oscillation signal, "there is no natural hierarchy of coupling among generations in the lepton sector". Moreover, at the same time the atmospheric neutrino anomaly indicates "the existence of an additional sterile neutrino state besides the three active flavor neutrino states".

The Majorana idea,[2] recently analyzed in detail by Ahluwalia,[3] gives alternative way of describing neutral particles, which is based on the treatment of self/anti-self charge conjugate states. This formalism is believed at the moment to be able to provide a natural mechanism of neutrino oscillations through the Majorana mass term in the Lagrangian.

In Ref. 3 in the framework of the Majorana–McLennan–Case kinematical scheme, the following type-II bispinors of the $(j,0)\oplus(0,j)$ representation space have been defined in the momentum representation:

$$\lambda(p^\mu)\equiv\begin{pmatrix}(\zeta_\lambda\Theta_{[j]})\phi_L^*(p^\mu)\\ \phi_L(p^\mu)\end{pmatrix}\,,\qquad \rho(p^\mu)\equiv\begin{pmatrix}\phi_R(p^\mu)\\ (\zeta_\rho\Theta_{[j]})^*\phi_R^*(p^\mu)\end{pmatrix}\,,\tag{1}$$

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where ζ_λ and ζ_ρ are the phase factors fixed by the conditions of self/anti-self charge conjugacy, $\Theta_{[j]}$ is the Wigner time-reversal operator for spin j. In this letter we show that the construction based on the type-II spinors leads to another example of the Nigam–Foldy–Bargmann–Wightman–Wigner (FNBWW) type quantum field theory.

The irreducible projective representations of the quantum-mechanical Poincaré group have been enumerated by Wigner.[4,5] He showed that one has to distinguish four cases. The Dirac field, that describes the eigenstates of the charge operator, belongs to the simplest one.[a] In the other three cases, there is a phenomenon which could be called as doubling of an ordinary Fock space (or, in the Schrödinger language, doubling the number of components of the wave function). An explicit example of the FNBWW-type quantum field theory has recently been presented[6] in the $(1,0)\oplus(0,1)$ representation of the extended Lorentz group (see also earlier papers Refs. 7–9). The remarkable feature of the construction presented in Ref. 6 is that in such a framework a boson and its antiboson have opposite intrinsic parities. In this letter we present a construction in which fermion and antifermion have the same intrinsic parity. We prove this by working out explicitly their properties under operators of discrete symmetries C, P and T.

Let us begin with the transformation properties of the left ϕ_L (and $\chi_L = (\zeta^*_\rho \Theta_{[j]})\phi^*_R$), and the right ϕ_R (and $\chi_R = (\zeta_\lambda \Theta_{[j]})\phi^*_L$) two-spinors. In particular, the $(1/2,0)$ spinors transform with respect to the restricted Lorentz transformations according to the Wigner's rules:

$$\phi_R(p^\mu) = \Lambda_R(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\phi_R(\overset{\circ}{p}{}^\mu) = \exp\left(+\frac{\boldsymbol{\sigma}\cdot\boldsymbol{\varphi}}{2}\right)\phi_R(\overset{\circ}{p}{}^\mu)\,, \tag{2a}$$

$$\chi_R(p^\mu) = \Lambda_R(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\chi_R(\overset{\circ}{p}{}^\mu) = \exp\left(+\frac{\boldsymbol{\sigma}\cdot\boldsymbol{\varphi}}{2}\right)\chi_R(\overset{\circ}{p}{}^\mu)\,, \tag{2b}$$

and the $(0,1/2)$ spinors,

$$\phi_L(p^\mu) = \Lambda_L(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\phi_L(\overset{\circ}{p}{}^\mu) = \exp\left(-\frac{\boldsymbol{\sigma}\cdot\boldsymbol{\varphi}}{2}\right)\phi_L(\overset{\circ}{p}{}^\mu)\,, \tag{3a}$$

$$\chi_L(p^\mu) = \Lambda_L(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\chi_L(\overset{\circ}{p}{}^\mu) = \exp\left(-\frac{\boldsymbol{\sigma}\cdot\boldsymbol{\varphi}}{2}\right)\chi_L(\overset{\circ}{p}{}^\mu)\,, \tag{3b}$$

where $\boldsymbol{\varphi}$ are the Lorentz boost parameters, e.g. Ref. 10, $\boldsymbol{\sigma}$ are the Pauli matrices. In the chiral representation, one can choose the spinorial basis (zero-momentum spinors) in the following way[b]:

[a] Nevertheless, let us not forgetting that the Dirac construction allows one to describe both particle and its antiparticle which have opposite eigenvalues of the charge operator.

[b] Overall phase factors of left- and right-spinors are assumed to be the same, see Eqs. (22a) and (22b) in Ref. 3c. In this letter, we try to keep the notation of the cited reference.

$$\lambda^S_\uparrow(\overset{\circ}{p}{}^\mu)=\sqrt{\frac{m}{2}}\begin{pmatrix}0\\ i\\ 1\\ 0\end{pmatrix},\qquad \lambda^S_\downarrow(\overset{\circ}{p}{}^\mu)=\sqrt{\frac{m}{2}}\begin{pmatrix}-i\\ 0\\ 0\\ 1\end{pmatrix},$$

$$\lambda^A_\uparrow(\overset{\circ}{p}{}^\mu)=\sqrt{\frac{m}{2}}\begin{pmatrix}0\\ -i\\ 1\\ 0\end{pmatrix},\qquad \lambda^A_\downarrow(\overset{\circ}{p}{}^\mu)=\sqrt{\frac{m}{2}}\begin{pmatrix}i\\ 0\\ 0\\ 1\end{pmatrix},\tag{4a}$$

$$\rho^S_\uparrow(\overset{\circ}{p}{}^\mu)=\sqrt{\frac{m}{2}}\begin{pmatrix}1\\ 0\\ 0\\ -i\end{pmatrix},\qquad \rho^S_\downarrow(\overset{\circ}{p}{}^\mu)=\sqrt{\frac{m}{2}}\begin{pmatrix}0\\ 1\\ i\\ 0\end{pmatrix},$$

$$\rho^A_\uparrow(\overset{\circ}{p}{}^\mu)=\sqrt{\frac{m}{2}}\begin{pmatrix}1\\ 0\\ 0\\ i\end{pmatrix},\qquad \rho^A_\downarrow(\overset{\circ}{p}{}^\mu)=\sqrt{\frac{m}{2}}\begin{pmatrix}0\\ 1\\ -i\\ 0\end{pmatrix}.\tag{4b}$$

The indices $\uparrow\downarrow$ should be referred to as the chiral helicity quantum number introduced in Ref. 3. Using the boost (2a)–(3b) the reader would immediately find the four-spinors of the second kind $\lambda^{S,A}_{\uparrow\downarrow}(p^\mu)$ and $\rho^{S,A}_{\uparrow\downarrow}(p^\mu)$ in an arbitrary frame:

$$\lambda^S_\uparrow(p^\mu)=\frac{1}{2\sqrt{E+m}}\begin{pmatrix}ip_l\\ i(p^-+m)\\ p^-+m\\ -p_r\end{pmatrix},\qquad \lambda^S_\downarrow(p^\mu)=\frac{1}{2\sqrt{E+m}}\begin{pmatrix}-i(p^++m)\\ -ip_r\\ -p_l\\ (p^++m)\end{pmatrix},\tag{5a}$$

$$\lambda^A_\uparrow(p^\mu)=\frac{1}{2\sqrt{E+m}}\begin{pmatrix}-ip_l\\ -i(p^-+m)\\ (p^-+m)\\ -p_r\end{pmatrix},\qquad \lambda^A_\downarrow(p^\mu)=\frac{1}{2\sqrt{E+m}}\begin{pmatrix}i(p^++m)\\ ip_r\\ -p_l\\ (p^++m)\end{pmatrix},\tag{5b}$$

$$\rho^S_\uparrow(p^\mu)=\frac{1}{2\sqrt{E+m}}\begin{pmatrix}p^++m\\ p_r\\ ip_l\\ -i(p^++m)\end{pmatrix},\qquad \rho^S_\downarrow(p^\mu)=\frac{1}{2\sqrt{E+m}}\begin{pmatrix}p_l\\ (p^-+m)\\ i(p^-+m)\\ -ip_r\end{pmatrix},\tag{5c}$$

$$\rho^A_\uparrow(p^\mu) = \frac{1}{2\sqrt{E+m}} \begin{pmatrix} p^+ + m \\ p_r \\ -ip_l \\ i(p^+ + m) \end{pmatrix}, \qquad \rho^A_\downarrow(p^\mu) = \frac{1}{2\sqrt{E+m}} \begin{pmatrix} p_l \\ (p^- + m) \\ -i(p^- + m) \\ ip_r \end{pmatrix}, \tag{5d}$$

with $p_r = p_x + ip_y$, $p_l = p_x - ip_y$, $p^\pm = p_0 \pm p_z$. Therefore, one has [Eqs. (48a) and (48b) of Ref. 3c]

$$\rho^S_\uparrow(p^\mu) = -i\lambda^A_\downarrow(p^\mu)\,, \qquad \rho^S_\downarrow(p^\mu) = +i\lambda^A_\uparrow(p^\mu)\,, \tag{6a}$$

$$\rho^A_\uparrow(p^\mu) = +i\lambda^S_\downarrow(p^\mu)\,, \qquad \rho^A_\downarrow(p^\mu) = -i\lambda^S_\uparrow(p^\mu)\,. \tag{6b}$$

The normalization of the spinors $\lambda^{S,A}_{\uparrow\downarrow}(p^\mu)$ and $\rho^{S,A}_{\uparrow\downarrow}(p^\mu)$ are as follows:

$$\bar\lambda^S_\uparrow(p^\mu)\lambda^S_\downarrow(p^\mu) = -im\,, \qquad \bar\lambda^S_\downarrow(p^\mu)\lambda^S_\uparrow(p^\mu) = +im\,, \tag{7a}$$

$$\bar\lambda^A_\uparrow(p^\mu)\lambda^A_\downarrow(p^\mu) = +im\,, \qquad \bar\lambda^A_\downarrow(p^\mu)\lambda^A_\uparrow(p^\mu) = -im\,, \tag{7b}$$

$$\bar\rho^S_\uparrow(p^\mu)\rho^S_\downarrow(p^\mu) = +im\,, \qquad \bar\rho^S_\downarrow(p^\mu)\rho^S_\uparrow(p^\mu) = -im\,, \tag{7c}$$

$$\bar\rho^A_\uparrow(p^\mu)\rho^A_\downarrow(p^\mu) = -im\,, \qquad \bar\rho^A_\downarrow(p^\mu)\rho^A_\uparrow(p^\mu) = +im\,. \tag{7d}$$

All the other conditions are equal to zero (provided that $\vartheta_1^{L,R} + \vartheta_2^{L,R} = \pi$).

First of all, one must deduce equations for the Majorana-like spinors in order to see what dynamics do the neutral particles have. Obviously it is difficult to build the Lagrangian dynamics from Eqs. (30) and (31) of Ref. 3c (they are very unwieldy). Nevertheless, one can use another generalized form of the Ryder–Burgard relation (cf. Eq. (26) of Ref. 3c and Ref. 11) for zero-momentum spinors:

$$\left[\phi^h_L(\overset{\circ}{p}{}^\mu)\right]^* = (-1)^{1/2-h}\, e^{-i(\vartheta_1^L + \vartheta_2^L)}\, \Theta_{[1/2]}\, \phi^{-h}_L(\overset{\circ}{p}{}^\mu)\,. \tag{8}$$

Relations for zero-momentum right spinors are obtained with the substitution $L \leftrightarrow R$. h is the helicity quantum number for the left and right two-spinors. Hence, implying that $\lambda^S(p^\mu)$ (and $\rho^A(p^\mu)$) for positive-frequency solutions; $\lambda^A(p^\mu)$ (and $\rho^S(p^\mu)$), for negative-frequency solutions, one can deduce the dynamical coordinate-space equations[11c]

$$i\gamma^\mu\partial_\mu\lambda^S(x) - m\rho^A(x) = 0\,, \tag{9a}$$

$$i\gamma^\mu\partial_\mu\rho^A(x) - m\lambda^S(x) = 0\,, \tag{9b}$$

$$i\gamma^\mu\partial_\mu\lambda^A(x) + m\rho^S(x) = 0\,, \tag{9c}$$

$$i\gamma^\mu\partial_\mu\rho^S(x) + m\lambda^A(x) = 0\,. \tag{9d}$$

They can be written in the eight-component form as follows:

$$[i\Gamma^\mu\partial_\mu - m]\Psi_{(+)}(x) = 0\,, \tag{10a}$$

$$[i\Gamma^\mu\partial_\mu + m]\Psi_{(-)}(x) = 0\,, \tag{10b}$$

with

$$\Psi_{(+)}(x) = \begin{pmatrix}\rho^A(x)\\ \lambda^S(x)\end{pmatrix}, \qquad \Psi_{(-)}(x) = \begin{pmatrix}\rho^S(x)\\ \lambda^A(x)\end{pmatrix}, \quad \text{and} \quad \Gamma^\mu = \begin{pmatrix}0 & \gamma^\mu\\ \gamma^\mu & 0\end{pmatrix}. \tag{11}$$

One can also rewrite the equations into the two-component form. Similar formulations have been presented by M. Markov,[12] and A. Barut and G. Ziino.[13]

The Dirac-like and Majorana-like field operators can be built from both $\lambda^{S,A}(p^\mu)$ and $\rho^{S,A}(p^\mu)$, or their combinations (see Eqs. (46), (47) and (49) in Ref. 3c). For instance,

$$\Psi(x^\mu) \equiv \int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_p}\sum_\eta \left[\lambda^S_\eta(p^\mu)a_\eta(\mathbf{p})\exp(-ip\cdot x) + \lambda^A_\eta(p^\mu)b^\dagger_\eta(\mathbf{p})\exp(+ip\cdot x)\right]\,. \tag{12}$$

Operators of discrete symmetries (charge conjugation and space inversion) are given by

$$\begin{aligned} S^c_{[1/2]} &= e^{i\vartheta^c_{[1/2]}}\begin{pmatrix}0 & i\Theta_{[1/2]}\\ -i\Theta_{[1/2]} & 0\end{pmatrix}\mathcal{K} = \mathcal{C}_{[1/2]}\mathcal{K}\,,\\ S^s_{[1/2]} &= e^{i\vartheta^s_{[1/2]}}\begin{pmatrix}0 & \mathbb{1}_2\\ \mathbb{1}_2 & 0\end{pmatrix} = e^{i\vartheta^s_{[1/2]}}\gamma^0\,. \end{aligned} \tag{13}$$

In the Fock space, operations of the charge conjugation and space inversions can be defined through unitary operators such that:

$$\begin{aligned} U^c_{[1/2]}\Psi(x^\mu)(U^c_{[1/2]})^{-1} &= \mathcal{C}_{[1/2]}\Psi^\dagger_{[1/2]}(x^\mu)\,,\\ U^s_{[1/2]}\Psi(x^\mu)(U^s_{[1/2]})^{-1} &= \gamma^0\Psi(x'^\mu)\,, \end{aligned} \tag{14}$$

the time reversal operation, through *an antiunitary* operator[c]

$$\left[V^{\mathrm{T}}_{[1/2]}\Psi(x^\mu)(V^{\mathrm{T}}_{[1/2]})^{-1}\right]^\dagger = S(T)\Psi^\dagger(x''^\mu)\,, \tag{15}$$

with $x'^\mu \equiv (x^0,-\mathbf{x})$ and $x''^\mu = (-x^0,\mathbf{x})$. We further assume the vacuum state to be assigned an even P- and C-eigenvalue and, then, proceed as in Ref. 6.

[c]Let us remind that the operator of hermitian conjugation does not act on c-numbers on the left of Eq. (15). This fact is connected with the properties of an antiunitary operator:

$$\left[V^{\mathrm{T}}\lambda A(V^{\mathrm{T}})^{-1}\right]^\dagger = \left[\lambda^* V^{\mathrm{T}}A(V^{\mathrm{T}})^{-1}\right]^\dagger = \lambda\left[V^{\mathrm{T}}A^\dagger(V^{\mathrm{T}})^{-1}\right].$$

As a result, we have the following properties of creation (annihilation) operators in the Fock space:

$$U^s_{[1/2]} a_\uparrow(\mathbf{p}) (U^s_{[1/2]})^{-1} = -i a_\downarrow(-\mathbf{p}) \,, \qquad U^s_{[1/2]} a_\downarrow(\mathbf{p}) (U^s_{[1/2]})^{-1} = +i a_\uparrow(-\mathbf{p}) \,, \quad (16a)$$

$$U^s_{[1/2]} b^\dagger_\uparrow(\mathbf{p}) (U^s_{[1/2]})^{-1} = +i b^\dagger_\downarrow(-\mathbf{p}) \,, \qquad U^s_{[1/2]} b^\dagger_\downarrow(\mathbf{p}) (U^s_{[1/2]})^{-1} = -i b_\uparrow(-\mathbf{p}) \,, \quad (16b)$$

which signifies that the states created by the operators $a^\dagger(\mathbf{p})$ and $b^\dagger(\mathbf{p})$ have very different properties with respect to the space inversion operation, comparing with the Dirac states (also mentioned in Ref. 13):

$$U^s_{[1/2]} |\mathbf{p}, \uparrow\rangle^+ = +i|-\mathbf{p}, \downarrow\rangle^+ \,, \qquad U^s_{[1/2]} |\mathbf{p}, \uparrow\rangle^- = +i|-\mathbf{p}, \downarrow\rangle^- \,, \quad (17a)$$

$$U^s_{[1/2]} |\mathbf{p}, \downarrow\rangle^+ = -i|-\mathbf{p}, \uparrow\rangle^+ \,, \qquad U^s_{[1/2]} |\mathbf{p}, \downarrow\rangle^- = -i|-\mathbf{p}, \uparrow\rangle^- \,. \quad (17b)$$

For the charge conjugation operation in the Fock space we have two physically different possibilities. The first one, e.g.,

$$U^c_{[1/2]} a_\uparrow(\mathbf{p}) (U^c_{[1/2]})^{-1} = +b_\uparrow(\mathbf{p}) \,, \qquad U^c_{[1/2]} a_\downarrow(\mathbf{p}) (U^c_{[1/2]})^{-1} = +b_\downarrow(\mathbf{p}) \,, \quad (18a)$$

$$U^c_{[1/2]} b^\dagger_\uparrow(\mathbf{p}) (U^c_{[1/2]})^{-1} = -a^\dagger_\uparrow(\mathbf{p}) \,, \qquad U^c_{[1/2]} b^\dagger_\downarrow(\mathbf{p}) (U^c_{[1/2]})^{-1} = -a^\dagger_\downarrow(\mathbf{p}) \,, \quad (18b)$$

in fact, has some similarities with the Dirac construction. The actions of this operator on the physical states are

$$U^c_{[1/2]} |\mathbf{p}, \uparrow\rangle^+ = +|\mathbf{p}, \uparrow\rangle^- \,, \qquad U^c_{[1/2]} |\mathbf{p}, \downarrow\rangle^+ = +|\mathbf{p}, \downarrow\rangle^- \,, \quad (19a)$$

$$U^c_{[1/2]} |\mathbf{p}, \uparrow\rangle^- = -|\mathbf{p}, \uparrow\rangle^+ \,, \qquad U^c_{[1/2]} |\mathbf{p}, \downarrow\rangle^- = -|\mathbf{p}, \downarrow\rangle^+ \,. \quad (19b)$$

But, one can construct the charge conjugation operator in the Fock space which acts, e.g., in the following manner:

$$\tilde{U}^c_{[1/2]} a_\uparrow(\mathbf{p}) (\tilde{U}^c_{[1/2]})^{-1} = -b_\downarrow(\mathbf{p}) \,, \qquad \tilde{U}^c_{[1/2]} a_\downarrow(\mathbf{p}) (\tilde{U}^c_{[1/2]})^{-1} = -b_\uparrow(\mathbf{p}) \,, \quad (20a)$$

$$\tilde{U}^c_{[1/2]} b^\dagger_\uparrow(\mathbf{p}) (\tilde{U}^c_{[1/2]})^{-1} = +a^\dagger_\downarrow(\mathbf{p}) \,, \qquad \tilde{U}^c_{[1/2]} b^\dagger_\downarrow(\mathbf{p}) (\tilde{U}^c_{[1/2]})^{-1} = +a^\dagger_\uparrow(\mathbf{p}) \,, \quad (20b)$$

and, therefore,

$$\tilde{U}^c_{[1/2]} |\mathbf{p}, \uparrow\rangle^+ = -|\mathbf{p}, \downarrow\rangle^- \,, \qquad \tilde{U}^c_{[1/2]} |\mathbf{p}, \downarrow\rangle^+ = -|\mathbf{p}, \uparrow\rangle^- \,, \quad (21a)$$

$$\tilde{U}^c_{[1/2]} |\mathbf{p}, \uparrow\rangle^- = +|\mathbf{p}, \downarrow\rangle^+ \,, \qquad \tilde{U}^c_{[1/2]} |\mathbf{p}, \downarrow\rangle^- = +|\mathbf{p}, \uparrow\rangle^+ \,. \quad (21b)$$

Investigations of several important cases, which are different from the above, are discussed in a separate paper. Next, by straightforward verification one can convince oneself about the assertions made in Refs. 3 and 14 (see also Ref. 7) are correct as it is possible for the operators of the space inversion and charge conjugation to commute with each other in the Fock space. For instance,

$$U^c_{[1/2]} U^s_{[1/2]} |\mathbf{p}, \uparrow\rangle^+ = +i U^c_{[1/2]} |-\mathbf{p}, \downarrow\rangle^+ = +i|-\mathbf{p}, \downarrow\rangle^- \,, \tag{22a}$$

$$U^s_{[1/2]} U^c_{[1/2]} |\mathbf{p}, \uparrow\rangle^+ = U^s_{[1/2]} |\mathbf{p}, \uparrow\rangle^- = +i|-\mathbf{p}, \downarrow\rangle^- \,. \tag{22b}$$

The second choice of the charge conjugation operator answers for the case when the $\tilde{U}^c_{[1/2]}$ and $U^s_{[1/2]}$ operations anticommute:

$$\tilde{U}^c_{[1/2]} U^s_{[1/2]} |\mathbf{p}, \uparrow\rangle^+ = +i \tilde{U}^c_{[1/2]} |-\mathbf{p}, \downarrow\rangle^+ = -i|-\mathbf{p}, \uparrow\rangle^- \,, \tag{23a}$$

$$U^s_{[1/2]} \tilde{U}^c_{[1/2]} |\mathbf{p}, \uparrow\rangle^+ = -U^s_{[1/2]} |\mathbf{p}, \downarrow\rangle^- = +i|-\mathbf{p}, \uparrow\rangle^- \,. \tag{23b}$$

Next, one can compose states which would have somewhat similar properties to those which we have become accustomed. The states $|\mathbf{p}, \uparrow\rangle^+ \pm i|\mathbf{p}, \downarrow\rangle^+$ answer for positive (negative) parity, respectively. But, what is important, is that *the anti-particle states* (moving backward in time) have the same properties with respect to the operation of space inversion as the corresponding *particle states* (as opposed to $j = 1/2$ Dirac particles). This is again in accordance with the analysis of Nigam and Foldy and Ahluwalia. The states which are eigenstates of the charge conjugation operator in the Fock space are

$$U^c_{[1/2]} \big(|\mathbf{p}, \uparrow\rangle^+ \pm i|\mathbf{p}, \uparrow\rangle^- \big) = \mp i \big(|\mathbf{p}, \uparrow\rangle^+ \pm i|\mathbf{p}, \uparrow\rangle^- \big) \,. \tag{24}$$

There is no simultaneous set of states which were "eigenstates" of the operator of the space inversion and of the charge conjugation $U^c_{[1/2]}$.

Finally, the time reversal *anti-unitary* operator in the Fock space should be defined in such a way that the formalism is compatible with the CPT theorem. If we want the Dirac states to transform as $V(T)|\mathbf{p}, \pm 1/2\rangle = \pm|-\mathbf{p}, \mp 1/2\rangle$, we have to choose (within a phase factor), Ref. 15:

$$S(T) = \begin{pmatrix} \Theta_{[1/2]} & 0 \\ 0 & \Theta_{[1/2]} \end{pmatrix} \,. \tag{25}$$

Thus, in the first relevant case we obtain for the $\Psi(x^\mu)$ field,[d] Eq. (12):

$$V^{\mathrm{T}} a^\dagger_\uparrow(\mathbf{p}) (V^{\mathrm{T}})^{-1} = a^\dagger_\downarrow(-\mathbf{p}) \,, \qquad V^{\mathrm{T}} a^\dagger_\downarrow(\mathbf{p}) (V^{\mathrm{T}})^{-1} = -a^\dagger_\uparrow(-\mathbf{p}) \,, \tag{26a}$$

$$V^{\mathrm{T}} b_\uparrow(\mathbf{p}) (V^{\mathrm{T}})^{-1} = b_\downarrow(-\mathbf{p}) \,, \qquad V^{\mathrm{T}} b_\downarrow(\mathbf{p}) (V^{\mathrm{T}})^{-1} = -b_\uparrow(-\mathbf{p}) \,. \tag{26b}$$

To summarize, we note that we have constructed another explicit example of the Bargmann–Wightman–Wigner theory. The matters of physical dynamics connected

[d]In connection with the proposal of the eight-component equation we still note that some modifications in arguments concerning the formalism for time-reversal operation are possible.

with this mathematical construction should be solved in future taking into account the gauge interactions with potential fields[11c] and the experimental setup.

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