DOME: Discrete oriented muon emission in GEANT4 simulations

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Abstract

Amongst various applications that experience a multi-directional particle source is the muon scattering tomography where a number of horizontal detectors of a limited angular acceptance conventionally track the cosmic-ray muons. In this study, we exhibit an elementary strategy that might be at disposal in diverse computational applications in the GEANT4 simulations with the purpose of hemispherical particle sources. To further detail, we initially generate random points on a spherical surface for a sphere of a practical radius by employing Gaussian distributions for the three components of the Cartesian coordinates, thereby obtaining a generating surface for the initial positions of the corresponding particles. Since we do not require the half bottom part of the produced spherical surface for our tomographic applications, we take the absolute value of the vertical component in the Cartesian coordinates by leading to a half-spherical shell, which is traditionally called a hemisphere. Last but not least, we direct the generated particles into the target material to be irradiated by favoring a selective momentum direction that is based on the vector construction between the random point on the hemispherical surface and the origin of the target material, hereby optimizing the particle loss through the source biasing. We also show a second scheme where the coordinate transformation is performed between the spherical coordinates and the Cartesian coordinates, and the above-mentioned procedure is applied to orient the generated muons towards the target material. In the end, a recipe hinged on the restrictive planes from our previous study is furthermore provided, and we incorporate our strategies by using G4ParticleGun in the GEANT4 code. While we plan to exert our strategy in the computational practices for muon scattering tomography, these source schemes might find its straightforward applications in different neighboring fields including but not limited to atmospheric sciences, space engineering, and astrophysics where a 3D particle source is a necessity for the modeling goals.

Keywords: Muon tomography; GEANT4; Monte Carlo simulations; Discretized energy spectra; Source biasing; Restrictive planes

1 Introduction

In practice, various shapes of radiation sources including but limited to planar surfaces and parabolic beams have been utilized to mimic the associated reality in the desired applications, and one of these geometries includes hemispherical surfaces [1]. In this study, we describe the implementation steps of two schemes by aiming at building a hemispherical muon source where the generated particles are oriented towards a specific point or plane that we call selective momentum direction. While there exist different schemes to generate the 2D/3D sources, we prefer to use the existing algorithms in GEANT4 [2], i.e. G4RandGauss :: shoot() and G4UniformRand() as a distribution function. Whereas the geometrical shape of the 2D/3D sources plays an important role or is a parameter for this aim, the momentum direction is another variable that awaits for a user decision. In this study, we first generate a spherical surface by using three Gaussian distributions for the three components of the Cartesian coordinates and we direct the generated particles from their initial positions on this spherical surface to the preferred location(s) by using a vector construction as described in our previous study [3]. This methodology is called discrete oriented muon emission (DOME) where the kinetic energy of the generated particles is intentionally discrete for the computational purposes as already implemented in another study [4]. In the latter scheme, we generate the initial positions by randomizing the spherical variables, i.e. altitude and longitude, and we perform the coordinate transformation from the spherical coordinates to the Cartesian coordinates [5–7]. We repeat the same operations as performed in the first scheme. This study is organized as follows. Section 2 describes the first scheme that is hinged on the Gaussian distribution functions, while section 3 consists of the second methodology founded on the coordinate transformation from the spherical coordinates to the Cartesian coordinates. Whereas an alternative focusing scheme is explained in section 4, we draw our conclusions in section 5.

2 Central focus scheme

2.1 Generation trough Gaussian distributions

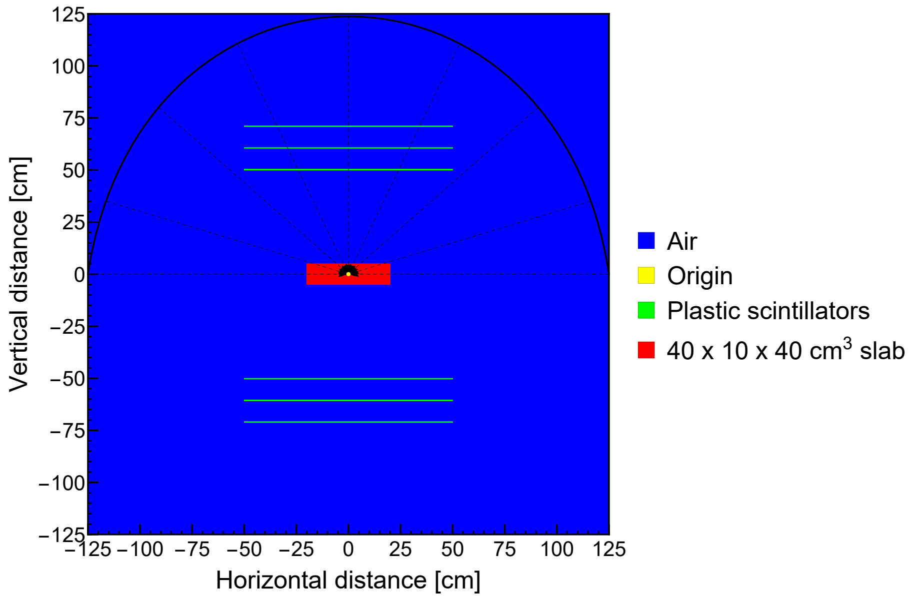

Figure 1: Delineation of the generated particles from the hemispherical source with a momentum direction towards the origin.

Our objective is to build a hemispherical muon source that surrounds our tomographic setup [8] similar to the other tomographic configurations existing in the literature [9–11] as delineated in Fig. 1. On the first basis, the particle locations in the Cartesian coordinates are generated by using the Gaussian distributions formally defined as G4RandGauss::shoot() in GEANT4 as written in

$$x_0 = G(\bar{x}, \sigma_x, x) = \text{G4RandGauss :: shoot()} \tag{1}$$

and

$$y_0 = G(\bar{y}, \sigma_y, y) = \text{G4RandGauss :: shoot()} \tag{2}$$

and

$$z_0 = G(\bar{z}, \sigma_z, z) = \text{G4RandGauss :: shoot()} \tag{3}$$

where $\bar{x} = \bar{y} = \bar{z} = 0$ and $\sigma_x = \sigma_y = \sigma_z = 1$ by definition. The generated spatial points are renormalized in order to form a unit sphere as indicated in

$$x_0^* = \frac{x_0}{\sqrt{x_0^2 + y_0^2 + z_0^2}}, \quad y_0^* = \frac{y_0}{\sqrt{x_0^2 + y_0^2 + z_0^2}}, \quad z_0^* = \frac{z_0}{\sqrt{x_0^2 + y_0^2 + z_0^2}} \tag{4}$$

Given a sphere of radius denoted by R, the initial positions on the spherical surface of radius R in cm in the Cartesian coordinates are obtained as follows

$$x_i = R * x_0^*, \quad y_i = R * |y_0^*| = R * \text{ABS}(y_0^*), \quad z_i = R * z_0^* \tag{5}$$

where the y-component of the Cartesian coordinates constituting the vertical axis is positively defined in order to yield the hemispherical surface. Then, the generated particles on the spherical surface are directed to the origin

$$x_f = 0, \quad y_f = 0, \quad z_f = 0 \tag{6}$$

Then, by constructing a vector from the hemispherical surface to the origin, one obtains

$$px = x_f - x_i, \quad py = y_f - y_i, \quad pz = z_f - z_i \tag{7}$$

Thus, the selective momentum direction denoted by $\vec{P} = (P_x, P_y, P_z)$ is

$$P_x = \frac{px}{\sqrt{px^2 + py^2 + pz^2}}, \quad P_y = \frac{py}{\sqrt{px^2 + py^2 + pz^2}}, \quad P_z = \frac{pz}{\sqrt{px^2 + py^2 + pz^2}} \tag{8}$$

2.2 Generation via coordinate transformation

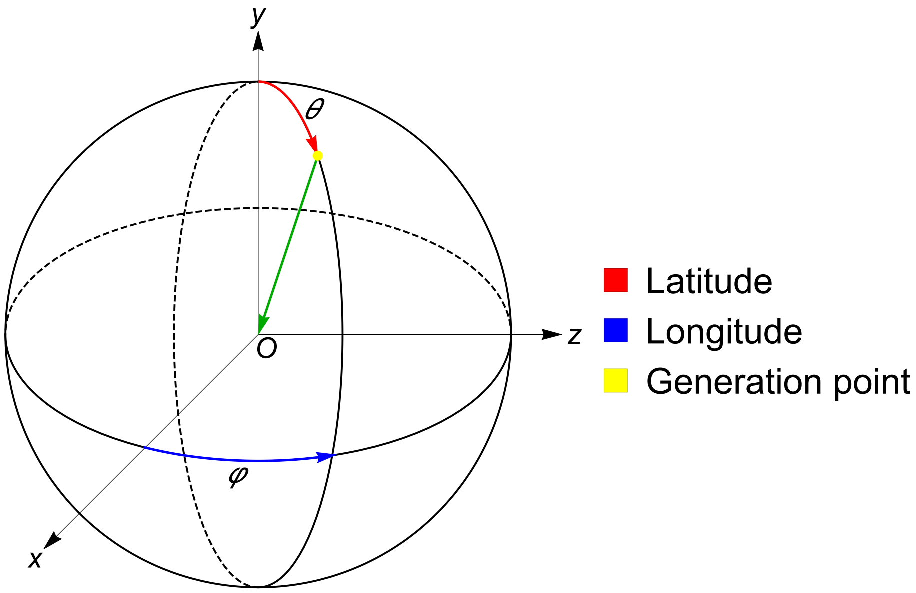

Figure 2: Spherical variables consisting of latitude denoted by θ and longitude indicated by φ with respect to the Cartesian coordinates (x,y,z).

The latter scheme is composed of the coordinate transformation as depicted in Fig. 2. To begin with, two numbers, i.e. q_1 and q_2, are uniformly generated to be inserted to the associated expression of the spherical variables as follows

$$q_1 = \text{G4UniformRand()} \tag{9}$$

and

$$q_2 = \text{G4UniformRand()} \tag{10}$$

The surface generation is initiated by randomizing θ as well as φ as shown in

$$\theta = \arccos(2 \times q_1 - 1) \tag{11}$$

and

$$\varphi = 2 \times \pi \times q_2 \tag{12}$$

The coordinate transformation yields the generated points on the hemispherical surface for a sphere of radius R in the Cartesian coordinates as described in

$$x_i = R \times \cos\theta \times \cos\varphi \tag{13}$$

and

$$y_i = R \times |\sin\theta| = R \times \mathrm{ABS}(\sin\theta) \tag{14}$$

and

$$z_i = R \times \cos\theta \times \sin\varphi \tag{15}$$

where the y-component of the Cartesian coordinates constituting the vertical axis is repeatedly positively defined in order to yield the hemispherical surface as usual. Then, the generated particles on the spherical surface are again directed to the origin

$$x_f = 0, \quad y_f = 0, \quad z_f = 0 \tag{16}$$

Then, by constructing a vector from the hemispherical surface to the origin, one obtains

$$px = x_f - x_i, \quad py = y_f - y_i, \quad pz = z_f - z_i \tag{17}$$

Thus, the selective momentum direction denoted by $\vec{P} = (P_x, P_y, P_z)$ is

$$P_x = \frac{px}{\sqrt{px^2 + py^2 + pz^2}}, \quad P_y = \frac{py}{\sqrt{px^2 + py^2 + pz^2}}, \quad P_z = \frac{pz}{\sqrt{px^2 + py^2 + pz^2}} \tag{18}$$

Finally, the simulation preview through either scheme is displayed in Fig. 3.

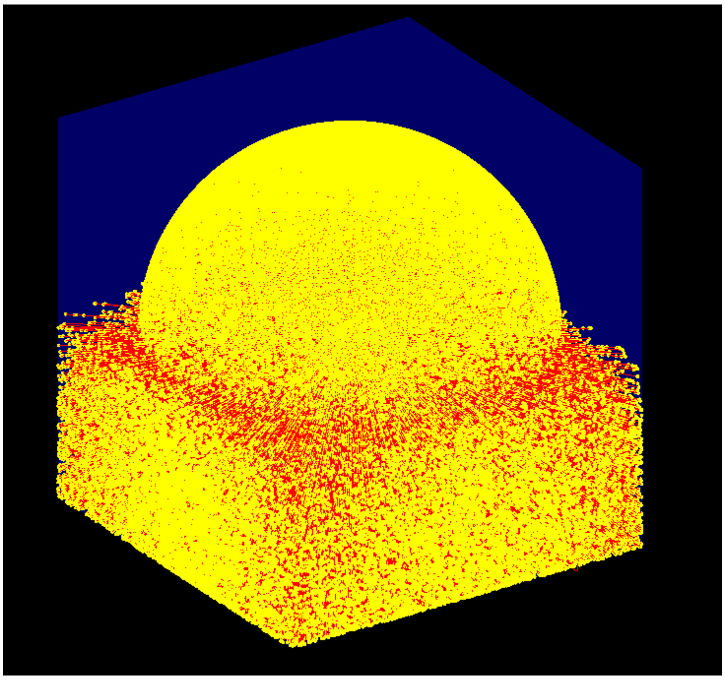

Figure 3: Hemispherical muon source in GEANT4.

3 Restrictive planar focus scheme

The generated particles from any initial point on the hemispherical surface might be directed to a location randomly selected on a pseudo plane that restricts the momentum direction, which also leads to the minimization of the particle escape. Thus, the particle locations in cm on a restrictive plane of $2L \times 2D$ cm^2 situated at y=0 are supposed to have the spatial coordinates such that

$$x_f = -L + 2 \times L \times \mathrm{G4UniformRand}(), \quad y_f = 0, \quad z_f = -D + 2 \times D \times \mathrm{G4UniformRand}() \tag{19}$$

Then, by constructing a vector from the generative hemispherical surface to the restrictive plane, one obtains

$$px = x_f - x_i, \quad py = y_f - y_i, \quad pz = z_f - z_i \tag{20}$$

Thus, the selective momentum direction, i.e. $\vec{P} = (P_x, P_y, P_z)$, is

$$P_x = \frac{px}{\sqrt{px^2 + py^2 + pz^2}}, \quad P_y = \frac{py}{\sqrt{px^2 + py^2 + pz^2}}, \quad P_z = \frac{pz}{\sqrt{px^2 + py^2 + pz^2}} \tag{21}$$

4 Conclusion

In this study, we explore the posibility to use the random number generators that are already defined in the GEANT4 code. By profiting from these random number generators, we provide a number of source schemes where the first strategy is based on the Gauss distributions, whereas the latter procedure requires a coordinate transformation by utilizing the spherical variables. Finally, we obtain a hemispherical muon source where the kinetic energies of the generated muons are discretized, and the momentum directions of these generated muons are selective by means of the vector constructions. We call this source discrete oriented muon emission (DOME).

References

[1] D. Pagano, G. Bonomi, A. Donzella, A. Zenoni, G. Zumerle, and N. Zurlo, "Ecomug: an efficient cosmic muon generator for cosmic-ray muon applications," *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, vol. 1014, p. 165732, 2021.

[2] S. Agostinelli, J. Allison, K. a. Amako, J. Apostolakis, H. Araujo, P. Arce, M. Asai, D. Axen, S. Banerjee, G. Barrand, *et al.*, "Geant4—a simulation toolkit," *Nuclear instruments and methods in physics research section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, vol. 506, no. 3, pp. 250–303, 2003.

[3] A. I. Topuz, M. Kiisk, and A. Giammanco, "Particle generation trough restrictive planes in geant4 simulations for potential applications of cosmic ray muon tomography," *arXiv preprint arXiv:2201.07068*, 2022.

[4] A. I. Topuz and M. Kiisk, "Towards energy discretization for muon scattering tomography in geant4 simulations: A discrete probabilistic approach," *arXiv preprint arXiv:2201.08804*, 2022.

[5] G. Marsaglia, "Choosing a point from the surface of a sphere," *The Annals of Mathematical Statistics*, vol. 43, no. 2, pp. 645–646, 1972.

[6] Y. Tashiro, "On methods for generating uniform random points on the surface of a sphere," *Annals of the Institute of Statistical Mathematics*, vol. 29, no. 1, pp. 295–300, 1977.

[7] E. W. Weisstein, ""disk point picking." from mathworld-a wolfram web resource," *http://mathworld. wolfram.com/*, 2011.

[8] A. Georgadze, M. Kiisk, M. Mart, E. Avots, and G. Anbarjafari, "Method and apparatus for detection and/or identification of materials and of articles using charged particles," Jan. 7 2021. US Patent App. 16/977,293.

[9] K. N. Borozdin, G. E. Hogan, C. Morris, W. C. Priedhorsky, A. Saunders, L. J. Schultz, and M. E. Teasdale, "Radiographic imaging with cosmic-ray muons," *Nature*, vol. 422, no. 6929, pp. 277–277, 2003.

[10] L. Frazão, J. Velthuis, S. Maddrell-Mander, and C. Thomay, "High-resolution imaging of nuclear waste containers with muon scattering tomography," *Journal of Instrumentation*, vol. 14, no. 08, p. P08005, 2019.

[11] L. Frazão, J. Velthuis, C. Thomay, and C. Steer, "Discrimination of high-z materials in concrete-filled containers using muon scattering tomography," *Journal of Instrumentation*, vol. 11, no. 07, p. P07020, 2016.