100 years of gravitational waves: the event GW150914 and the future of gravitational theories

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Abstract

Did the event GW150914 ultimately confirm the general theory of relativity (GTR)? Here we show that the correct answer is *not yet* and we discuss the future of gravitational theories in the framework of gravitational wave (GW) astronomy. In particular, we show that $f(R)$ theories of gravity are ruled out with a confidence level higher than 90%.

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For a long time, the discovery of GW emissions from the compact binary system with two neutron stars PSR1913+16 [1] has been the ultimate motivation for the design, implementation, and advancement of extremely sophisticated GW detection technology. Physicists working in this field of research need this technology to conduct thorough investigations of GWs in order to advance science. The observation of GWs from a binary black hole (BH) merger (event GW150914) [2], which occurred in the 100th anniversary of Albert Einstein's prediction of GWs [3], has recently shown that this ambitious challenge has been won. The event GW150914 represented a cornerstone for science and for gravitational physics in particular. In fact, this remarkable event equipped scientists with the means to give definitive proof of the existence of GWs, the existence of BHs having mass greater than 25 solar masses and the existence of binary systems of BHs which coalesce in a time less than the age of the Universe [2]. A subsequent analysis of GW150914 constrained the graviton Compton wavelength of those alternative theories of gravity (ATG) in which the graviton is massive and placed a level of 90% confidence on the lower bound of 10^{13} km [4]. Within their statistical uncertainties, the LIGO Scientific Collaboration and the Virgo Collaboration have not found evidence of violations of the GTR in the genuinely strong-field regime of gravity [4]. On the other hand, the possibility that ATG are still alive after the event GW150914 has been emphasized in [5]. In fact, in [6] two important questions have been raised, verbatim: "*Does gravity really behave as predicted by Einstein in the vicinity of black holes, where the fields are very strong? Can dark energy and the acceleration of the Universe be explained if we modify Einstein's gravity?*" The current situation is that "*We are only just beginning to answer these questions*" [6].

Among the various kinds of ATG, $f(R)$ theories and scalar tensor gravity (STG) seem to be the most popular among gravitational physicists [7, 8, 9, 12]. These theories attempt to extend the framework of the GTR by modifying the Lagrangian, with respect to the standard Einstein-Hilbert gravitational Lagrangian, through the addition of high-order terms in the curvature invariants (terms like R^2, $R^{ab}R_{ab}$, $R^{abcd}R_{abcd}$, $R\Box R$, $R\Box^k R$) and/or terms with scalar fields non-minimally coupled to geometry (terms like $\phi^2 R$) [7, 8, 9, 12]. In this letter we will focus on these two classes of ATG. Criticisms on such theories arises from the fact that lots of them can be excluded by requirements of cosmology and solar system tests [9, 11, 15]. Thus, one needs the additional assumption that the variation from the standard GTR must be weak [12].

For the following discussion the key point is that STG and $f(R)$ theories have an additional GW polarization which, in general, is massive with respect to the two standard polarizations of the GTR; see [10, 11, 12, 13, 14]. As GW detection is performed in a laboratory environment on Earth, one typically uses the coordinate system in which space-time is locally flat and the distance between any two points is given simply by the difference in their coordinates in the sense of Newtonian physics. This is the so-called gauge of the local observer [10, 13, 14, 16]. In such a gauge the GWs manifest themselves by exerting tidal forces on the masses (the mirror and the beam-splitter in the case of an interferometer) [10, 13, 14, 16]. By putting the beam-splitter in the origin of the coordinate system, the components of the separation vector are the coordinates of the mirror. The effect of the GW is to drive the mirror to have oscillations [10, 13, 14, 16]. Let us consider a mirror that has the initial (unperturbed) coordinates x_{M0}, y_{M0} and z_{M0} , where there is a GW propagating in the z direction.

In the GTR the GW admits only the standard $+$ and $\times$ polarizations [10, 16]. We label the respective metric perturbations as h_+ and $h_\times$. To the first order approximation of h_+ and $h_\times$ the motion of the mirror due to the GW is (we work with $16\pi G = 1$, $c = 1$ and $\hbar = 1$ in the following) [10, 16]

$$\begin{aligned} x_M(t) &= x_{M0} + \tfrac{1}{2}[x_{M0}h_+(t) - y_{M0}h_\times(t)] \\ y_M(t) &= y_{M0} - \tfrac{1}{2}[y_{M0}h_+(t) + x_{M0}h_\times(t)] \\ z_M(t) &= z_{M0}. \end{aligned} \tag{1}$$

STG can be massless [10, 12, 14]. In this case, calling h_Φ the metric perturbation due to the additional GW polarization, to the first order approximation of h_+, $h_\times$ and h_Φ the motion of the mirror due to the GW is [10, 14]

$$\begin{aligned} x_M(t) &= x_{M0} + \tfrac{1}{2}[x_{M0}h_+(t) - y_{M0}h_\times(t)] + \tfrac{1}{2}x_{M0}h_\Phi(t) \\ y_M(t) &= y_{M0} - \tfrac{1}{2}[y_{M0}h_+(t) + x_{M0}h_\times(t)] + \tfrac{1}{2}y_{M0}h_\Phi(t) \\ z_M(t) &= z_{M0}. \end{aligned} \tag{2}$$

$f(R)$ theories are generally massive [11, 12, 13, 14]. The cases of massive STG and massive $f(R)$ theories are totally equivalent [11, 12, 13, 14]. This is not surprising because it is well known that there is a more general conformal equivalence between $f(R)$ theories and STG [7, 12]. Again, we call h_Φ the metric perturbation due to the additional GW polarization. To the first order approximation of h_+, $h_\times$ and h_Φ the motion of the mirror due to the GW in massive STG and massive $f(R)$ theories is [12, 13, 14]

$$\begin{aligned} x_M(t) &= x_{M0} + \tfrac{1}{2}[x_{M0}h_+(t) - y_{M0}h_\times(t)] + \tfrac{1}{2}x_{M0}h_\Phi(t) \\ y_M(t) &= y_{M0} - \tfrac{1}{2}[y_{M0}h_+(t) + x_{M0}h_\times(t)] + \tfrac{1}{2}y_{M0}h_\Phi(t) \\ z_M(t) &= z_{M0} + \tfrac{1}{2}z_{M0}\tfrac{m^2}{\omega^2}h_\Phi(t), \end{aligned} \tag{3}$$

where m and ω are the mass and the frequency of the GW's third massive mode, which is interpreted in terms of a wave packet [12, 13, 14]. The presence of the little mass m implies that the speed of the third massive mode is less than the speed of light; this generates the longitudinal component and drives the mirror oscillations of the z direction [11, 13, 14], which is shown by the third of eqs. (3).

The analysis of the LIGO Scientific Collaboration and the Virgo Collaborations implies $m \leq 1.2 \times 10^{-22} \frac{eV}{c^2}$ (in standard units) in eqs. (3) with 90%

confidence [4]. In other words, we can assume $m \simeq 0$ in eqs. (3) with 90% confidence. Thus, with 90% confidence, it follows that $f(R)$ theories and STG must be massless for all purposes. This implies that eqs. (3) reduce to eqs. (2) with 90% confidence. We know that STG can be massless [10, 12, 14]. Thus, let us see what happens in the case of massless $f(R)$ theories, that, to our knowledge, has not been analysed in the literature. In order to linearize the $f(R)$ theories one uses the identifications [11]

$$\Phi \rightarrow \frac{df(R)}{dR} \qquad \text{and} \qquad \frac{dV}{d\Phi} \rightarrow \frac{2f(R)-R\frac{df(R)}{dR}}{3}. \tag{4}$$

The mass of the GW is given by [11]

$$\frac{dV}{d\Phi} \simeq m^2 \delta\Phi, \tag{5}$$

where $\delta\Phi$ is the variation of the effective scalar field Φ near a minimum for the effective potential V, see [11] for details. Thus, for $m \simeq 0$ one gets

$$2f(R) \simeq R\frac{df(R)}{dR}. \tag{6}$$

By separating the variables eq. (6) is easily solved as

$$f(R) \simeq R^2. \tag{7}$$

But we recall that the class of αR^n theories (where n is not restricted to be an integer and $\alpha > 0$ has the dimensions of a mass squared [17]), is viable only for $n = 1 + \varepsilon$ with $0 \leq \varepsilon \ll 1$ [15, 16, 17]. Consequently, since the R^2 theory is not viable, we've discovered an interesting result: with a confidence level higher than 90%, ALL $f(R)$ theories are ruled out. In fact, on one hand, all massive $f(R)$ theories are ruled out by the analysis in [4] with 90% confidence. On the other hand, massless $f(R)$ theories are ruled out by our previous analysis.

In any case, we observe that the GTR is not yet ultimately confirmed by the results of the LIGO Scientific Collaboration and the Virgo Collaborations in [2, 4]. In fact, on one hand, 90% confidence is not 100% confidence and this implies that there is still room for massive ATG with 10% confidence. In this case, the oscillations of the interferometer's mirror will be governed by eqs. (3). On the other hand, there is room for massless STG. In fact, the analysis of the LIGO Scientific Collaboration and the Virgo Collaborations in [4] did not put constraints on the massless STG. In this latter case, the oscillations of the interferometer's mirror will be governed by eqs. (2). Thus, we understand which is the key point here. Only a perfect knowledge of the motion of the interferometer's mirror will permit one to determine if the GTR is the definitive theory of gravity. In order to ultimately conclude that the GTR is the definitive theory of gravity, one must prove that the oscillations of the interferometer's mirror are in fact governed by eqs. (1). Otherwise, if one proves that the oscillations of the interferometer's mirror are in fact governed by eqs. (2) or eqs. (3), then the GTR must be extended. At the present time, the sensitivity of the current ground based GW interferometers is not sufficiently high to determine if the oscillations of the interferometer's mirror are governed by eqs. (1), or if they are governed by eqs. (2) or eqs. (3). A network including interferometers with different orientations is indeed required and we're hoping that future advancements in ground-based projects and space-based projects will have a sufficiently high sensitivity. Such advancements would enable physicists to determine, with absolute precision, the direction of GW propagation and the motion of the various involved mirrors. In other words, in the nascent GW astronomy we hope not only to obtain new, precious astrophysical information, but we also hope to be able to discriminate between eqs. (1), eqs. (2), and eqs. (3). Such advances in GW technology would equip us with the means and results to ultimately confirm the GTR or, alternatively, to ultimately clarify that the GTR must be extended.

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