

# Initial Friedmann universe, its spatial flatness, matter creation, and the cosmological term

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## Abstract

In the Friedmann equations, an infinite initial density is avoided only when the universe is spatially flat. With such equations being then valid when the scale factor  $a = 0$ , the universe must also be in the state of vacuum when  $a$  is infinitesimal. Matter creation thus largely comes from nonlinearity of the Friedmann equations at a finite  $a$ . For the correspondence between such an initial vacuum and the Friedmann universe, therefore, a vacuum density is suggested to be gravitationally significant only when it could also have a matter phase. The effective cosmological term is then due to such a vacuum density.

## 1 Introduction

A major discrepancy between observed (presumable) consequences of the effective cosmological term in the Einstein equations of general relativity and a quantum-theory assessment of this term has been known as the (old) cosmological constant problem. Different aspects of this issue have been repeatedly discussed [1]. One recently emphasized additional element of such discussions is also order-of-magnitude observational coincidence of the energy density underlying the cosmological term with the matter energy density [1, 2, 3].

Quantum mechanical understanding of vacuum contributions to the cosmological term is a major fundamental issue [1, 4, 5, 6]. Quantum theory is however primarily applicable when a microscopic nature of matter is considered, as is statistical mechanics in particular. Although gravity is also expected to be present in microscopic processes, the most secured domain of application of the theory of general relativity is described by phenomena of an intrinsically macroscopic nature. In this sense, this theory is similar to the classical thermodynamics. Different aspects relating these two theories have indeed been under intensive discussion [7]. Respective theories of microscopic and macroscopic matter description have to be consistent with each other. At any energy scale, however, the universe as a whole is expected to remain infinite. Its macroscopic description thus has to be permitted for any value of the scale factor.

If the Einstein equations of general relativity are correct on the universe scale, therefore, the observations on the cosmological constant problems must have an interpretation in terms of these equations alone, for these observations may suggest their direct conflict with the Einstein theory. Such interpretations, however, seem to be often based only on a phenomenologically (including observationally) motivated specification of the spatial flatness and on additional phenomenological assumptions giving rise to a negative pressure in the Friedmann formulation for the Einstein equations. Apart from other effects [1], in particular, these assumptions come from models of holography [7], vacuum decay [8], bulk dissipation [9], or irreversible matter creation [10]. This note shows that a nonsingular and spatially flat universe with deflationary vacuum decay [8, 11, 12, 13] and an effective cosmological term generally arises from the Friedmann form of the Einstein equations without additional assumptions.

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## 2 Initial universe and its spatial flatness

### 2.1 Mathematical observations

For the Einstein equations [14] with the Friedmann-Robertson-Walker (FRW) metric [15], let us consider the acceleration and Friedmann equations (hereafter both referred to as the Friedmann equations) [1, 15, 16] (in the units with the speed of light  $c = 1$ ),

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p), \quad (1)$$

$$H^2 = \frac{8\pi G\rho}{3} - \frac{k}{a^2}. \quad (2)$$

Here  $a$  is the scale factor,  $H = \dot{a}/a$  is the Hubble parameter,  $G$  is the gravitation constant,  $k = 0, \pm 1$  is the space-curvature index,  $\rho$  and  $p$  stand for the overall energy density and pressure, respectively, and the dots denote the derivatives with respect to time,  $t$ .

Let  $\rho$  and  $p$  be viewed as the functions of  $a$ . Eqs. (1) and (2) are thus invariant with respect to both a shift in time (transformation  $t \mapsto t + \tau$  for any  $\tau$ ) and the direction of time (transformation  $\tau + t \mapsto \tau - t$  for any  $\tau$ ). Since  $a = 0$  is a minimal value of  $a$  when the latter is considered as a function of either the positive or the negative time, the existence of  $\dot{a}|_{a(\tau_0)=0}$  (regardless of how the time moment,  $\tau_0$ , at which  $a = 0$  is defined) then requires that  $\dot{a}|_{a(\tau_0)=0} = \dot{a}|_{t=\tau_0} = \dot{a}|_{t=\tau_0-0} = [da(\tau_0-t)/d(\tau_0-t)]|_{t=0} = -[da(\tau_0+t)/d(\tau_0+t)]|_{t=0} = -\dot{a}|_{t=\tau_0} = 0$ . Eq. (2) thus implies that the density at  $a(\tau_0) = 0$  could be finite [11, 12, 13, 17] only if the universe is spatially flat, consistently with the observations [1, 2, 3]. In particular, application of the l'Hospital rule for  $a \rightarrow 0$  to Eq. (2), along with Eq. (1), then suggests [allowing for  $d()/da = (\dot{\phantom{a}})/\dot{a}$ ] that the only equation of state permitted for  $a \rightarrow 0$  is that of vacuum:

$$p_v^0 = -\rho_v^0, \quad (3)$$

where  $\rho_v^0$  and  $p_v^0$  are the energy density and pressure when  $a \rightarrow 0$ , respectively. [Eq. (3) would not be affected if Eqs. (1) and (2) also had a cosmological constant term that is not a part of  $\rho$  and  $p$ . That such a term has to be irrelevant will be seen from below.] Along with Eq. (3), Eq. (1) thus implies that  $\ddot{a} > 0$  at  $a = 0$ , i.e. that  $a = 0$  is a minimum of  $a(t)$ .

After a medium with the equation of state described by Eq. (3) was first introduced in [4], such a medium has also been hypothesized in [11, 12, 13] to describe an initial state of the universe that does not meet the conditions for existence of a cosmological singularity [18]. Analogous possibilities for preventing the singularity also predicted in [18] to arise otherwise in the context of a black hole gravitational collapse have been considered in [19].

For a finite  $\rho(0)$  and Eqs. (1) and (2) being valid at  $a = 0$ , one could expect  $[d\rho/da]_{a \rightarrow 0} = O(a)$  to hold as well. Let us however also note that Eq. (2) formally requires that  $\rho(a) = \rho(-a)$ . Eq. (1) then implies the validity of  $p(a) = p(-a)$  as well. As for  $\dot{a}(\tau_0)$  above, this means that

$$\left[\frac{d^{(2n+1)}\rho(a)}{da^{(2n+1)}}\right]_{a=0} = \left[\frac{d^{(2n+1)}p(a)}{da^{(2n+1)}}\right]_{a=0} = 0, \quad n = 0, 1, 2, 3, \dots \quad (4)$$

have to hold. For  $n = 0$ , Eqs. (4) thus suggest that  $\rho(a)$  and  $p(a)$  are actually expected to have their respective extrema at  $a = 0$ . Eq. (3) could therefore also be obtained if one considers the  $a \rightarrow 0$  limit of the energy-conservation equation,

$$\frac{d\rho(a)}{da} = -3\frac{\rho(a) + p(a)}{a}. \quad (5)$$

This equation is implied by Eqs. (1) and (2) as well.

By differentiation of Eq. (5) and using Eq. (5) itself, one could then obtain:

$$\frac{d^2\rho(a)}{da^2} = -\frac{3}{a}\left\{\frac{d}{da}[\rho(a) + p(a)] + \frac{1}{3}\frac{d\rho(a)}{da}\right\}. \quad (6)$$

With the use of the l'Hospital rule in Eq. (6) for  $a \rightarrow 0$  as well as of the Eqs. (4) for  $n = 0$ ,

$$\left[\frac{d^2 p(a)}{da^2}\right]_{a \rightarrow 0} = -\frac{5}{3} \left[\frac{d^2 \rho(a)}{da^2}\right]_{a \rightarrow 0} \quad (7)$$

is thus derived. Since  $p(a) > -\rho(a)$  for  $a > 0$ , therefore, Eqs. (3), (4) for  $n = 0$ , (5), and (7) suggest that  $a = 0$  has to be a maximum of  $\rho(a)$  {i.e.,  $[d^2 \rho(a)/da^2]_{a \rightarrow 0} < 0$ , as implied by Eq. (5) for small  $|a| \geq 0$ } and a minimum of  $p(a)$  {i.e.,  $[d^2 p(a)/da^2]_{a \rightarrow 0} > 0$ }.

When Eqs. (1) and (2) are valid at  $a \rightarrow 0$ , their linearization with respect to  $a$  at  $a = 0$  is legitimate, say upon multiplication of Eq. (2) by  $a^2$  and of Eq. (1) by  $a$ . With  $[d\rho/da]_{a=0} = 0$ , such linearized Eq. (2) and Eq. (1) itself at  $a \rightarrow 0$  lead to the equation of state described by Eq. (3) being approximately valid for an infinitesimal  $a$  ( $\equiv a_i$  say) as well. In particular, the right hand side of the Eq. (5) multiplied by  $a$  differs from zero by a magnitude at least of the order of  $a^2$ , since  $d\rho(a)/da = [d^2 \rho(0)/da^2]a + o(a)$  for  $a \rightarrow 0$ .

## 2.2 Physical implications

With a finite initial density when the universe is spatially flat, the (adiabatic) thermodynamics incorporated in the Friedmann equations thus actually requires that any (relativistic and nonrelativistic) matter significantly manifested for a finite  $a > 0$  macroscopically behave for  $a \rightarrow 0$  in compliance with the vacuum equation of state. Matter creation could therefore be interpreted as a matter self-identification when  $a$  increases from 0. This process has to become particularly intensive when  $a$  reaches such a (finite) value as renders the linearized Friedmann equations inadequate and necessitates a manifestation of the nonlinear effects.

Since the Friedmann equations do not contain any quantum physics, which has been invoked for interpretations in [11, 20], the matter-creation process is describable in classical terms alone. Based on the energy conservation underlying Eq. (5), such a possibility has generally been emphasized in [11, 12]. Various phenomenologically constructed models of vacuum decay into matter have thus been considered [8, 12, 13]. Alternative classical interpretations of matter creation and acceleration have been based on models of other different aspects of thermodynamic irreversibility [7, 9, 10].

The physics of matter creation suggested in Sec. 2.1 above might however also be viewed as classical manifestation of a quantum phenomenon some of whose aspects have been discussed in [21]. Such a phenomenon could be associated with production of permanent (real) matter particles out of temporarily existing (virtual) ones when annihilation of the latter is prevented by the effect of space expansion. The (quantum) temporal violation of energy conservation by the time-energy uncertainty is then turned into a classical decrease of the energy density. This decrease could be offset by growth of the overall vacuum energy due to the expansion work. Such work is done against the vacuum pressure by the repulsive vacuum gravitation while it acts on the created matter.

It is interesting to note in this context that the two exponential terms arising from (either of) Eqs. (1) and (2) and the (nearly) vacuum initial equation of state [approximately described by Eq. (3)] at an infinitesimal  $a$  might also actually correspond to two directions of time [5, 22], with respect to which Eqs. (1) and (2) are symmetric. Indeed, the term with the plus sign in the exponent has to vanish if  $a$  is required to be zero when  $-t = -\infty$ . In this case,  $a$  grows with  $-t$  being increased due to the term with the minus sign in the exponent. This latter term then has to vanish if  $a$  is required to be zero when  $t = -\infty$ . Growth of  $a$  with  $t$  being increased is then due to the term with the plus sign in the exponent. With such an interpretation, virtual particle-antiparticle pairs at an infinitesimal  $a$  are separated by the two branches of the CPT symmetry to eventually give rise to two different universes dominated by real matter (where entropy increases with  $t$ ) and real antimatter (where entropy increases with  $-t$ ), respectively [5, 22, 23].

A matterless vacuum with any homogeneous energy density could thus correspond to a macroscopic manifestation of the zero- $a$  limit of a Friedmann universe with the respective matter-density distribution,  $\rho_m(a) = \rho(a)$ , for a large enough  $a$ . Such a  $\rho_m(a)$  is expected to be the larger the higher its vacuum density is at  $a \rightarrow 0$ . From this perspective, matter does not have to disappear in the respective vacuum. The behavior of its particles is however expected to become such as mimicks the behavior of virtual particles in vacuum [11].

With the above correspondence, suggested by the general relativity theory, a growing  $a > 0$  stands for both the transition from vacuum to matter and the gravitational action of the residual vacuum and the created matter. In this context, a permanent part of vacuum density, if any, could then have only a(n equal) constant space-time response, for such a part corresponds to  $a = 0$ . A vacuum density thus has to be gravitationally significant only when it could also have a respective matter phase. For the Einstein equations, this is discussed in Sec. 4 as well.

There thus seems to be no reason to *a priori* expect what is often referred to as "breakdown" of the theory of general relativity when  $a \rightarrow 0$ . As discussed above, the universe remains a(n infinite) macroscopic object that is capable of maintaining a finite density in this limit as well. Since the universe has to be spatially flat to avoid an infinite initial density, the so-called "flatness problem" [24] does not have to exist either. In particular, this could question some of the cosmological motivation for introduction of an inflation paradigm into the early universe [15, 25]. Such a paradigm also seems to be inconsistent with the physics of matter creation outlined above. The latter process starts as soon as  $a > 0$ , when  $[d\rho(a)/da]a < 0$  in Eq. (5). Anticipated difficulties for reconciling the inflation paradigm with such theories as general relativity and thermodynamics have already been previously emphasized [26, 27].

## 3 Matter creation

### 3.1 General considerations

The observed accelerated expansion [1, 2, 3] is often associated with the presence of a new form of energy density other than a matter density in the universe. According to the standard ( $\Lambda$ CDM) scenario [1, 28], such a hypothetic density also has to be especially dominant for  $a \rightarrow \infty$ . In particular, Eq. (2) suggests that the overall density of the universe could be greater than zero at  $a \rightarrow \infty$  only when  $\dot{a}|_{a \rightarrow \infty} \rightarrow \infty$  as well, consistently with the standard scenario. (The velocity of space expansion is not limited by the speed of light.) Using the l'Hospital rule in Eq. (2) with  $\dot{a}|_{a \rightarrow \infty} \rightarrow \infty$  and Eq. (1) when  $a \rightarrow \infty$ , one then obtains:

$$p_v^\infty = -\rho_v^\infty, \quad (8)$$

where  $\rho_v^\infty$  and  $p_v^\infty$  are, respectively, the vacuum energy density and pressure at  $a \rightarrow \infty$ .

In the absence of phase transitions between vacuum and matter, the energy density satisfying Eq. (8) still has to be independent of the scale factor, including  $a \rightarrow \infty$ . This follows from Eq. (5) for such a density component and from more general thermodynamic considerations [29]. Had such phase transitions ended before the stage of accelerated expansion, therefore, the observed acceleration would have been driven by the vacuum density identified by Eq. (8). Such a scenario, however, implies the existence of a permanent part of the vacuum density that affects the universe dynamics without being the origin of matter. This would be inconsistent with gravitational significance of only such a vacuum density as could also have a respective matter phase. Such inconsistency is due to the correspondence between a homogeneous matterless vacuum and the respective Friedmann universe, as discussed in Sec. 2.2 above. In the framework of this correspondence, therefore, vacuum decay has to be active at the acceleration stage, as in some phenomenological models of this process and interpretations of cosmological data [8, 30]. Even if the rate of vacuum decay into matter is such as a gravitationally significant vacuum density would remain at any finite  $a$ , the argument just used still implies  $\rho_v^\infty = p_v^\infty = 0$ . The zero overall density and pressure at  $a \rightarrow \infty$  could not thus be avoided.

In the framework of Eqs. (1) and (2), emergence of the main part of the overall amount of current matter out of  $\rho_v^0$  is possible only when the scale factor is larger than a finite value,  $a_0$ . With such nonlinear matter creation being most intensive initially, a stage of dominance of matter attractive gravitation [1, 2, 3] could thus exist. To overcome this stage, the matter created by such a nonlinearity then has to emerge with a sufficiently high initial velocity of expansion. The corresponding initial value problem for subsequent evolution of the universe could thus become similar to the boundary value problem with a given overall (current) amount of matter in the standard cosmology [1, 28]. For the cosmological scenario considered herein, however, the high initial velocity of expansion acquires a simple and natural explanation.

As the scale factor,  $a$  has to remain physically meaningful even when it is infinitesimal, i.e. say when  $a = a_i$ , and an equation of state closely approximating Eq. (3) is macroscopically maintained in the universe. Matter could then be viewed as mainly test particles having only a higher-order infinitesimal effect on the equation of state, as seen from Eq. (5). Such an effect is however sufficient to allow a further growth of  $a$  and then a substantial manifestation of the phase transitions significantly changing the equation of state when the scale-factor magnitude triggering nonlinearity of the Friedmann equations,  $a = a_0$ , is reached.

For the reasons discussed above, the existence of a gravitationally significant part of the initial vacuum density is prevented so long as such a part is not the origin of any matter. Since  $\rho_v^0$  at  $a = 0$  thus results from the zero- $a$  limit of all the matter that would eventually be created (at an  $a \gg a_0$ ), one could write:

$$\rho_v^0 = \rho_{vr}^0 + \rho_{vn}^0. \quad (9)$$

Here  $\rho_{vr}^0$  and  $\rho_{vn}^0$  are the constituents of  $\rho_v^0$  that give rise to what is manifested at  $a > 0$  (particularly for  $a \geq a_0$ ) as radiation and nonrelativistic matter, respectively. The symbols in Eq. (9) thus represent the values of respective vacuum densities  $\rho_{vr}(a)$ ,  $\rho_{vn}(a)$ , and  $\rho_v(a) = \rho_{vr}(a) + \rho_{vn}(a)$  at  $a = 0$ . Such a separation between  $\rho_{vr}(a)$  and  $\rho_{vn}(a)$  in the vacuum state at  $a = a_v$  (including  $a_v = 0$ ), however, does not imply the absence of interaction between the respective components of matter after either one of them has or they both have been created at  $a = a_v + \delta a > a_v$  for a small enough  $\delta a$ .

### 3.2 Phase transitions and the effective vacuum density

Since a substantial density of matter begins to arise only at  $a = a_0$ , the corresponding moment of time could thus be associated with what is commonly referred to as the Big Bang. Although it could formally take an infinite time to reach  $a_0$  from an infinitesimal  $a$ , such a reference may be justified as marking the stage of a major transition of  $\rho_v^0$  to matter. [As mentioned in Sec. 2.1 above, Eqs. (1) and (2) are invariant with respect to a shift in time. It still seems to be most consistent with the common perception to associate both the Big Bang and the zero time with the beginning of a major transition of vacuum to matter [8, 12], which is identified by manifestation of the nonlinearity.]

Let  $\rho_r(a)$  and  $\rho_n(a)$  be the respective energy densities of radiation and nonrelativistic matter. Let us also allow for the equations of state for radiation,  $p_r(a) = (1/3)\rho_r(a)$ , and for nonrelativistic matter,  $p_n(a) = 0$ , where  $p_r(a)$  and  $p_n(a)$  are the pressures of radiation and nonrelativistic matter, respectively. One can then use additivity of the overall density and pressure as well:  $\rho(a) = \rho_v(a) + \rho_m(a)$  and  $p(a) = p_v(a) + p_m(a)$ , where  $p_v(a) = -\rho_v(a)$ ,  $\rho_m(a) = \rho_r(a) + \rho_n(a)$ , and  $p_m(a) = p_r(a) + p_n(a) [= p_r(a)]$ . Let the value of  $a$  at which vacuum decay (and the expansion acceleration) has to end be also denoted as  $a_m \leq \infty$ . As discussed in Sec. 3.1 above, therefore,  $\rho_v(a_m) = 0$ . With Eq. (9), integration of Eq. (5) from  $a = 0$  to  $a = a_m$  thus leads to:

$$\rho_v^0 = \rho_{vr}^0 + \rho_{vn}^0 = 4 \int_0^{a_m} \frac{\rho_r(a)}{a} da + 3 \int_0^{a_m} \frac{\rho_n(a)}{a} da + \rho_m(a_m). \quad (10)$$

For  $a_1 < a_m$ , however, Eq. (10) could be rewritten as

$$\begin{aligned} \rho_v^0 &= \int_0^{a_1} \frac{4\rho_r(a) + 3\rho_n(a)}{a} da + \int_{a_1}^{a_m} \frac{4\rho_r(a) + 3\rho_n(a)}{a} da + \rho_m(a_m) = \\ &\rho_v^0 - \rho_v(a_1) - \rho_m(a_1) + \rho_m(a_m) + \int_{a_1}^{a_m} \frac{4\rho_r(a) + 3\rho_n(a)}{a} da, \end{aligned} \quad (11)$$

where integration of Eq. (5) from 0 to  $a_1$  has been used to calculate the first integral in Eq. (11). At  $a = a_1$ , the effective vacuum density  $\rho_v^e(a_1) = \rho_v(a_1)$  could thus be written as

$$\rho_v^e(a_1) = \int_{a_1}^{a_m} \frac{4\rho_r(a) + 3\rho_n(a)}{a} da + \rho_m(a_m) - \rho_m(a_1). \quad (12)$$

Since  $\rho_v^e(a_1)$  has to be positive, the two first (definitely positive) terms in the right hand side of Eq. (12) have to exceed  $\rho_m(a_1)$ . An additional implication of  $\rho_v^e(a_1) > 0$  for the properties of vacuum decay could be seen from considering expansion of only such matter amount as is available at  $a = a_1$  (i.e., without an additional vacuum decay into matter for  $a > a_1$ ),  $\rho_m^1(a) \equiv \rho_r^1(a) + \rho_n^1(a)$ , where  $\rho_r^1(a) \equiv \rho_r(a_1)a_1^4/a^4$  is the respective radiation density, and  $\rho_n^1(a) \equiv \rho_n(a_1)a_1^3/a^3$  is such density of nonrelativistic matter.  $\rho_v^e(a_1) > 0$  thus implies that

$$\begin{aligned} \rho_v^e(a_1) &= \int_{a_1}^{a_m} \frac{4[\rho_r(a) - \rho_r^1(a)] + 3[\rho_n(a) - \rho_n^1(a)]}{a} da + \int_{a_1}^{a_m} \frac{4\rho_r^1(a) + 3\rho_n^1(a)}{a} da + \rho_m(a_m) - \rho_m(a_1) \\ &= \int_{a_1}^{a_m} \frac{\rho_r(a) - \rho_r^1(a)}{a} da + 3 \int_{a_1}^{a_m} \frac{\rho_m(a) - \rho_m^1(a)}{a} da + \rho_m(a_m) - \rho_m^1(a_m) > 0. \end{aligned} \quad (13)$$

Since  $\rho_m(a) > \rho_m^1(a)$  for  $a > a_1$ , only the first (integral) difference term in the last expression for the left hand side of inequality (13) may not be positive. This could happen if transition from radiation to nonrelativistic matter takes place and when such a transition is overall more effective than vacuum decay into radiation for  $a \in [a_1, a_m]$ . Even then, however, the other two (definitely positive) difference terms have to outweigh the first one to ensure  $\rho_v^e(a_1) > 0$ .

When no transition from radiation to nonrelativistic matter takes place, Eq. (13) for  $\rho_v^e(a_1)$  thus exposes the right hand side of Eq. (12) as being generated by a decay of the residual vacuum density between  $a = a_1$  and  $a = a_m$ . This vacuum density has been left after the preceding stages of phase transitions. These stages are characterized by transition from the initial vacuum phase of the whole matter [Eq. (9)] to the stage with the overall amount of both relativistic and nonrelativistic matter components at  $a = a_1$  apart from  $\rho_v^e(a_1)$ .

The expansion would be decelerating (accelerating) [1, 2, 3] at  $a = a_1$  if  $2\rho_v^e(a_1) < (>) \rho_m(a_1) + \rho_r(a_1) [= \rho_m(a_1) + 3p_m(a_1)]$ . In view of Eqs. (12) and (13), this is equivalent to

$$\int_{a_1}^{a_m} \frac{4\rho_r(a) + 3\rho_n(a)}{a} da + \rho_m(a_m) < (>) \frac{3\rho_m(a_1) + \rho_r(a_1)}{2} \iff \quad (14)$$

$$\int_{a_1}^{a_m} \frac{[\rho_r(a) - \rho_r^1(a)] + 3[\rho_m(a) - \rho_m^1(a)]}{a} da + \rho_m(a_m) - \rho_m^1(a_m) < (>) \frac{\rho_m(a_1) + \rho_r(a_1)}{2}. \quad (15)$$

Since matter creation has to be most intensive during the initial manifestation of the nonlinearity (when transition from radiation to nonrelativistic matter might also be taking place), the stage of nonlinear deceleration could be expected to eventually switch to that of such an acceleration. This corresponds to  $a_1$  moving from the initial to later nonlinear stages of universe evolution up until  $a_1 = a_m$ , where  $\rho_m(a_m) + \rho_r(a_m) > 0$  for  $a_m < \infty$  (if such a finite  $a_m$  exists) implies that a deceleration stage returns. The same order for  $2\rho_v^e(a_1)$  and  $\rho_m(a_1) + \rho_r(a_1)$  [and then also for  $\rho_v^e(a_1)$  and  $\rho_m(a_1)$ ] could thus be anticipated when both these magnitudes decrease with  $a_1$  increasing from such value of the scale factor as has made them equal. When  $a_1$  is the value of  $a$  relevant to the observations [1, 2, 3], the observed relation between  $\rho_v^e(a_1)$  and  $\rho_m(a_1)$  then provides another constraint for vacuum decay in terms of functions  $\rho_r(a)$  and  $\rho_n(a)$  for  $a \in [a_1, a_m]$  along with the ratio  $(a_m/a_1)$  in Eqs. (12) and (13).

## 4 The Einstein equations

The Einstein equations [14] with the effective cosmological term specified for the FRW metric by either of Eqs. (12) and (13) could thus be written as [signature  $(+, -, -, -)$  is assumed]

$$R_{ij} - \frac{1}{2}g_{ij}R = 8\pi GT_{ij}, \quad (16)$$

where  $g_{ij}$ ,  $R_{ij}$ , and  $T_{ij}$  ( $T_{i;j}^j = 0$ ) are the metric, Ricci, and total matter energy-momentum tensors, respectively, and  $R$  is the Ricci scalar curvature. When all phases of matter by definition described with  $T_{ij}$  are present,

$$T_{ij} = \tilde{T}_{ij} + g_{ij}\rho_v^e = g_{ij}(\rho_v^e - p_m) + (\rho_m + p_m)u_i u_j, \quad (17)$$

where  $\rho_v^e$  is the effective energy density of vacuum defined for the FRW  $a = a_1$  by either of Eqs. (12) and (13), and

$$\tilde{T}_{ij} = (\rho_m + p_m)u_i u_j - g_{ij}p_m \quad (18)$$

is the energy-momentum tensor of the matter phase. In Eqs. (17) and (18),  $\rho_m = \rho_r + \rho_n$  and  $p_m = p_r + p_n$  [ $p_r = (1/3)\rho_r$  and  $p_n = 0$ ] are the energy density and pressure, respectively, of the matter phase, and  $u_i$  is the four-velocity. Before the initial stage of phase transitions, only vacuum is present. In view of Eq. (9), Eq. (17) then has to look as

$$T_{ij} = g_{ij}\rho_v^0 = g_{ij}(\rho_{vr}^0 + \rho_{vn}^0). \quad (19)$$

If the right hand side of Eq. (9) also contained a part of the vacuum energy density that is not the origin of any matter, say  $\rho_v^v > 0$ , this vacuum density could not then stand for a phase of matter. Since  $\rho_v^v$  could thus be viewed as a constant of nature, Eqs. (17) and (19) legitimately imply that the corresponding term,  $g_{ij}\lambda$  with  $\lambda \equiv 8\pi G\rho_v^v$ , would then have to be present in the left hand side of Eqs. (16) as well. The possibility of having such a  $\lambda$ -term in the left hand side of Eqs. (16) is thus reserved for offsetting the effect of  $\rho_v^v$  in the right hand side of these equations.

Such a formulation of Eqs. (16) implies that a vacuum density matters gravitationally only when it arises from matter in the framework of these equations. The absence of  $\rho_v^v$  in Eqs. (17) and (19) is then due to the failure of vacuum with  $\rho_v^v$  to be the origin of matter. As discussed in Sec. 2.2 above, such a vacuum could only correspond to the Friedmann scale factor  $a = 0$ . This would be inconsistent with the growth of  $a > 0$  actually characterizing the expanding universe.  $\rho_v^v$  could not then be relevant to such a growth, for its space-time response also has to be constant when  $a$  is viewed as a measure of the phase transition as well. A  $\lambda$ -term,  $g_{ij}\lambda$ , in the left hand side of Eqs. (16) would also have to be only a(n equal) space-time response to the presence of  $8\pi Gg_{ij}\rho_v^v$  in the right hand side of these equations. The term  $8\pi Gg_{ij}\rho_v^v$  in Eqs. (16) thus has to either be zero or be canceled by its simultaneous presence in both sides of these equations.

## 5 Conclusions

It follows from the Friedmann equations that a universe being capable of avoiding an infinite initial energy density could only be spatially flat. Such equations with a finite initial density then have to be valid when the scale factor is equal to zero as well. They thus rigorously require that the corresponding universe be in the state of vacuum when the scale factor is infinitesimal. Matter creation is therefore largely the result of such a finite increase of the scale factor as necessitates the manifestation of respective nonlinear effects in the Friedmann equations. The beginning of the nonlinear stage of the matter-creating phase transitions could also be referred to as the Big Bang.

Correspondence between the homogeneous density of a matterless vacuum and a matter-density distribution in the respective Friedmann universe whose zero- $a$  limit is represented by such a vacuum is thus suggested by the Einstein equations. In the most general form, this correspondence implies that a vacuum density has to be gravitationally significant in these equations only when it could have a respective matter phase as well. It thus also means that vacuum decay into matter has to be continuing at the stage of accelerated expansion. The effective cosmological term is then specified by the vacuum phase of matter that has been left after the preceding phase transitions from vacuum to matter.

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