Large Deviations in Quantum Spin Chain

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Abstract

We show the full large deviation principle for KMS-states and C^*-finitely correlated states on a quantum spin chain. We cover general local observables. Our main tool is Ruelle's transfer operator method.

1 Introduction

While the large deviation for classical lattice spin systems constitutes a rather complete theory, our knowledge on large deviations in quantum spin systems is still restricted. Large deviation results for observables that depend only on one site were established in high temperature KMS-states, in [NR], using cluster expansion techniques. In [LR], large deviation upper bounds were proven for general observables, for KMS-states in the high temperature regime and in dimension one. Furthermore, it was shown that a state in one dimension, which satisfies a certain factorization property satisfies a large deviation upper bound [HMO]. This factorization property is satisfied by KMS-states as well as C^*-finitely correlated states. It was also shown in [HMO] that the distributions of the ergodic averages of a one-site observable with respect to an ergodic C^*-finitely correlated state satisfy full large deviation principle.

In spite of these progresses, the theory in quantum spin systems is not completed: we do not know if the large deviation lower bound holds for general observables, nor if the large deviation upper bound holds in the intermediate temperature KMS-states, for more than two dimensional spin systems. In this paper, we solve a part of the problem: we prove the full large deviation principle in dimension one.

The infinite spin chain with one site algebra $M_d(\mathbb{C})$ is given by the UHF C^*-algebra

$$\mathfrak{A}_{\mathbb{Z}} := \overline{\bigotimes_{\mathbb{Z}} M_d(\mathbb{C})}^{C^*},$$

which is the C^*- inductive limit of the local algebras

$$\left\{ \mathfrak{A}_\Lambda := \bigotimes_{\Lambda} M_d(\mathbb{C}) \middle| \quad \Lambda \subset \mathbb{Z}, \quad |\Lambda| < \infty \right\}.$$

For any subset S of $\mathbb{Z}$, we identify $\mathfrak{A}_S := \overline{\bigotimes_S M_d(\mathbb{C})}^{C^*}$ with a subalgebra of $\mathfrak{A}_{\mathbb{Z}}$ under the natural inclusion. The algebra of local observables is defined by

$$\mathfrak{A}_{loc} := \cup_{|\Lambda|<\infty} \mathfrak{A}_\Lambda.$$

Let $\gamma_j,\ j \in \mathbb{Z}$ be the lattice translation. A state ω is called translation-invariant if $\omega \circ \gamma_j = \omega$ for all $j \in \mathbb{Z}$. An interaction is a map Φ from the finite subsets of $\mathbb{Z}$ into $\mathfrak{A}_{\mathbb{Z}}$ such that $\Phi(X) \in \mathfrak{A}_X$ and $\Phi(X) = \Phi(X)^*$ for any finite $X \subset \mathbb{Z}$. In this paper, we will always assume that Φ is a finite range translation-invariant interaction, i.e., there exists $r \in \mathbb{N}$ such that

$$\Phi(X) = 0, \quad if \quad diam(X) > r,$$

and Φ is invariant under γ,

$$\Phi(X + j) = \gamma_j (\Phi(X)), \quad \forall j \in \mathbb{Z}, \quad \forall X \subset \mathbb{Z}.$$

For finite $\Lambda \subset \mathbb{Z}$, we set

$$H_\Phi(\Lambda) := \sum_{I \subset \Lambda} \Phi(I).$$

The distribution of $\frac{1}{n} H_\Phi([1,n])$ with respect to a state ω is the probability measure

$$\mu_n(B) := \omega(1_B(\frac{1}{n} H_\Phi([1,n]))), \quad B \in \mathcal{B},$$

where $\mathcal{B}$ denotes the Borel sets of $\mathbb{R}$ and $1_B(\frac{1}{n} H_\Phi([1,n])) \in \mathfrak{A}_{[1,n]}$ is the spectral projection of $\frac{1}{n} H_\Phi([1,n])$ corresponding to the set B.
Let $I : \mathcal{B} \to [0,\infty]$ be a lower semicontinuous mapping. We say that we have a large deviation upper bound for a closed set C if

$$\lim_{n\to\infty} \sup \frac{1}{n} \log \omega \left(1_C \left(\frac{1}{n} H_\Phi([1,n]) \right) \right) \leq - \inf_{x \in C} I(x).$$

Similarly, we have a large deviation lower bound for an open set O if

$$\lim_{n\to\infty} \inf \frac{1}{n} \log \omega \left(1_O \left(\frac{1}{n} H_\Phi([1,n]) \right) \right) \geq - \inf_{x \in O} I(x).$$

We say that $\{\mu_n\}$ satisfies the (full) large deviation principle if we have upper and lower bound for all closed and open sets, respectively. Furthermore, I is said to be a good rate function if all the level sets $\{x : I(x) \leq \alpha\}, \ \alpha \in [0, \infty)$ are compact subsets of $\mathbb{R}$(see [DZ]).

In this paper, we show the full large deviation principle for any kind of local observable, in KMS-states and C^*-finitely correlated states on quantum spin chain.

KMS-states Let Ψ be a translation-invariant finite range interaction, and define the finite volume Hamiltonian associated with a finite subset $\Lambda \subset \mathbb{Z}$ by

$$H_\Psi(\Lambda) := \sum_{I \subset \Lambda} \Psi(I).$$

It is known that there exists a strongly continuous one parameter group of $*$-automorphisms τ_Ψ on $\mathfrak{A}_\mathbb{Z}$, such that

$$\lim_{\Lambda \nearrow \mathbb{Z}} \left\| \tau_\Psi^t(A) - e^{itH_\Psi(\Lambda)} A e^{-itH_\Psi(\Lambda)} \right\| = 0, \quad \forall t \in \mathbb{R}, \quad \forall A \in \mathfrak{A}_\mathbb{Z}.$$

The equilibrium state corresponding to the interaction Ψ is characterized by the KMS condition. A state ω over $\mathfrak{A}_\mathbb{Z}$ is called a (τ_Ψ, β)-KMS state, if

$$\omega(A \tau_\Psi^{i\beta}(B)) = \omega(BA),$$

holds for any pair (A, B) of entire analytic elements for τ_Ψ. It is known that one dimensional quantum spin system has a unique (τ_Ψ, β)-KMS state for all $\beta \in \mathbb{R}$ [A1]. In this paper, we prove the large deviation principle for the (τ_Ψ, β)-KMS state:

Theorem 1.1 *Let Ψ be a translation-invariant finite range interaction and ω a (τ_Ψ, β)-KMS state. Furthermore, let Φ be another translation-invariant finite range interaction and $\mu_{n,\Phi}$ the distribution of $\frac{1}{n}H_\Phi([1,n])$ with respect to ω. Then the sequence $\{\mu_{n,\Phi}\}_{n\in\mathbb{N}}$ satisfies large deviation principle with a good rate function.*

Finitely correlated states The following recursive procedure to construct states on $\mathfrak{A}_\mathbb{Z}$ was introduced in [FNW], where the states obtained were called C^*-finitely correlated states. For the construction one needs a triple $(\mathcal{B}, \mathcal{E}, \rho)$, where $\mathcal{B}$ is a finite dimensional C^*-algebra, $\mathcal{E} : M_d(\mathbb{C}) \otimes \mathcal{B} \to \mathcal{B}$ a unital completely positive map and ρ a faithful state on $\mathcal{B}$ with density operator $\hat{\rho}$. Further, one has to assume that $\mathcal{E}$ and ρ are related so that $Tr_{M_d(\mathbb{C})}\mathcal{E}^*(\hat{\rho}) = \hat{\rho}$ holds. Then

$$\hat{\varphi}_1 := \mathcal{E}^*(\hat{\rho}); \quad \hat{\varphi}_n := \left(id_{M_d(\mathbb{C})}^{\otimes(n-1)} \otimes \mathcal{E}^*\right)\circ\cdots\circ\left(id_{M_d(\mathbb{C})} \otimes \mathcal{E}^*\right)\circ\mathcal{E}^*\left(\hat{\rho}\right);\ n = 2, 3, \cdots$$

defines a state on $M_d(\mathbb{C})^{\otimes n} \otimes \mathcal{B}$ for each $n \in \mathbb{N}$, and

$$\hat{\omega}_n := Tr_\mathcal{B}\hat{\varphi}_n$$

gives a state ω_n on $M_d(\mathbb{C})^{\otimes n}$. There exists a unique translation-invariant state ω with local restrictions $\omega|_{\mathfrak{A}_{[1,n]}} = \omega_n$. This is the C^*-finitely correlated state generated by $(\mathcal{B}, \mathcal{E}, \rho)$. In this paper, we prove large deviation principle for C^*-finitely correlated states:

Theorem 1.2 *Let ω be a C^*-finitely correlated state and Φ a translation-invariant finite range interaction. Let $\mu_{n,\Phi}$ be the distribution of $\frac{1}{n}H_\Phi([1,n])$ with respect to ω. Then the sequence $\{\mu_{n,\Phi}\}_{n\in\mathbb{N}}$ satisfies large deviation principle with a good rate function.*

In order to study the large deviations, we consider the corresponding logarithmic moment generating function, defined by

$$f(\alpha) = \lim_{n\to\infty} \frac{1}{n} \log \omega(e^{\alpha H_\Phi([1,n])}). \tag{1}$$

Theorem 1.3 (Gärtner-Ellis) *Let $\{\mu_n\}_{n\in\mathbb{N}}$ be a sequence of probability measures on the Borel sets of $\mathbb{R}$. Assume that the limit*

$$f(\alpha) := \lim_{n\to\infty} \frac{1}{n} \log \int e^{n\alpha x} d\mu_n(x)$$

exists and is differentiable for all $\alpha \in \mathbb{R}$. Let

$$I(x) := \sup_{\alpha\in\mathbb{R}} \{\alpha x - f(\alpha)\}.$$

Then $\{\mu_n\}$ satisfies the large deviation principle, i.e., we have

$$\lim_{n\to\infty} \sup \frac{1}{n} \log \mu_n(C) \leq - \inf_{x\in C} I(x),$$

and

$$\lim_{n\to\infty}\inf\frac{1}{n}\log\mu_n(O)\geq-\inf_{x\in O}I(x),$$

for any closed set C and any open set O, respectively. Furthermore, I is a good rate function.

In this paper, we use this Theorem to prove the large deviation principle, i.e., we prove the existence and differentiability of the logarithmic moment generating function $f(\alpha)$ (1). Our main tool is the transfer operator technique introduced by D.Ruelle for classical spin systems [R]. H. Araki applied this method to quantum spin systems and showed the analyticity of the mean free energy [A1]. This paper is basically an extension of this result. The non-commutative Ruelle transfer operator was further generalized in [GN] and [M]. We take advantage of these extensions.

The structure of this paper is as follows. In Section 2, we present a brief introduction to the non-commutative Ruelle transfer operator technique. In Section 3 and Section 4, we prove the large deviation principle, for KMS-states and C^*-finitely correlated states, respectively.

2 Non-commutative Ruelle transfer operator

In this section, we give a brief introduction of non-commutative Ruelle transfer operators studied in [A1] [GN] and [M]. We represent a generalized form but it can be proven in the same way as in [M]. We follow the notation in [M] and consider one-sided infinite system $\mathfrak{A}_{[1,\infty)}$. We also introduce a finite dimensional C^*-algebra $\mathcal{B}$. By $Q^{(j)},\ j\in\mathbb{N}$, we denote the element of $1_{\mathcal{B}}\otimes\mathfrak{A}_{[1,\infty)}$ with Q in the jth component of the tensor product of $\mathfrak{A}_{[1,\infty)}$ and the unit in any other component. Similarly, by $Q^{(0)}$ we denote an element in $\mathcal{B}\otimes 1_{\mathfrak{A}_{[1,\infty)}}$. We introduce a C^*-algebra

$$\mathcal{O}:=\left(\mathcal{B}\otimes\mathfrak{A}_{[1,\infty)}\right)\otimes\left(\mathcal{B}\otimes\mathfrak{A}_{[1,\infty)}\right)$$

and consider automorphisms $\{\Theta_j\}_{j\in\mathbb{N}}$ of $\mathcal{O}$ determined by

$$\Theta_j\left(Q^{(k)}\otimes 1\right)=\begin{cases}1\otimes Q^{(k)}, & for\ k\geq j\\ Q^{(k)}\otimes 1, & for\ k<j,\end{cases}$$

$$\Theta_j\left(1\otimes Q^{(k)}\right)=\begin{cases}Q^{(k)}\otimes 1, & for\ k\geq j\\ 1\otimes Q^{(k)}, & for\ k<j\end{cases}.$$

For any element Q in $\mathcal{B}\otimes\mathfrak{A}_{[1,\infty)}$, we set

$$\mathrm{var}_j(Q):=\left\|\Theta_j(Q\otimes 1)-Q\otimes 1\right\|,\quad j\in\mathbb{N}.$$

For any θ satisfying $0<\theta<1$ and $Q\in\mathcal{B}\otimes\mathfrak{A}_{[1,\infty)}$, we set

$$\|Q\|_\theta:=\max\left\{\frac{\mathrm{var}_jQ}{\theta^j},\quad j\in\mathbb{N}\right\}.$$

By F_θ we denote the dense subalgebra of $\mathcal{B} \otimes \mathfrak{A}_{[1,\infty)}$ consisting of elements Q with finite $\|Q\|_\theta$, and introduce the norm $|||Q|||$ of F_θ via the following equation:

$$|||Q||| = \max\{\|Q\|, \|Q\|_\theta\}.$$

F_θ is complete in this norm.
We need the $*$-isomorphism τ_{c+}, (resp. τ_{c-}) of $\mathcal{B} \otimes \mathfrak{A}_{[2,\infty)} \backsimeq \mathcal{B} \otimes 1_{\mathfrak{A}_{\{1\}}} \otimes \mathfrak{A}_{[2,\infty)}$ onto $\mathcal{B} \otimes \mathfrak{A}_{[1,\infty)}$ (resp. $\mathfrak{A}_{(-\infty,-2]} \otimes 1_{\mathfrak{A}_{\{-1\}}} \otimes \mathcal{B} \backsimeq \mathfrak{A}_{(-\infty,-2]} \otimes \mathcal{B}$ onto $\mathfrak{A}_{(-\infty,-1]} \otimes \mathcal{B}$) determined by

$$\tau_{c+}\left(x \otimes id_{\mathfrak{A}\{1\}} \otimes y\right) = x \otimes y,$$

for all $x \in \mathcal{B}$ and $y \in \mathfrak{A}_{[1,\infty)}$, (resp.

$$\tau_{c-}\left(y \otimes id_{\mathfrak{A}\{-1\}} \otimes x\right) = y \otimes x,$$

for all $x \in \mathcal{B}$ and $y \in \mathfrak{A}_{(-\infty,-1]}$.)

We now introduce a Ruelle transfer operator L:

Assumption 2.1 *Let a be an element in $\mathfrak{A}_{[1,\infty)}$, and*

$$\mathcal{E} : \mathcal{B} \otimes M_d(\mathbb{C}) \to \mathcal{B}$$

a completely positive unital map. Define a Ruelle transfer operator L on $\mathcal{B} \otimes \mathfrak{A}_{[1,\infty)}$ by

$$L(Q) := \tau_{c,+}\left(\mathcal{E} \otimes id_{[2,\infty)}\right)(a^* Q a), \quad Q \in \mathcal{B} \otimes \mathfrak{A}_{[1,\infty)}. \tag{2}$$

Assume that

(i) *The element a is in $F_\theta \cap \mathfrak{A}_{[1,\infty)}$ and invertible.*

(ii) *There exists an invariant state φ of L.*

(iii) *There exists a positive constant K such that the following bound is valid: Let Q be any strictly positive element in $\mathcal{B} \otimes (\mathfrak{A}_{loc} \cap \mathfrak{A}_{[1,\infty)})$. There exists a positive integer $N = N(Q)$ satisfying*

$$L^n(Q) \leq K \inf L^n(Q), \quad \forall n \geq N.$$

If Assumption 2.1 is valid, the restriction of L to the Banach space F_θ gives a bounded operator on F_θ. Assumption 2.1 guarantees the following properties of L.

Theorem 2.1 *Let L be a Ruelle transfer operator satisfying Assumption 2.1. Then*

(i) *There exists an element h in F_θ and a positive constant $m > 0$ such that*

$$L(h) = h, \quad m \leq h, \quad \varphi(h) = 1.$$

(ii) *Define an operator L_h and a state φ_h by*

$$L_h(Q) := h^{-\frac{1}{2}} L\left(h^{\frac{1}{2}} Q h^{\frac{1}{2}}\right) h^{-\frac{1}{2}}, \quad Q \in \mathcal{B} \otimes \mathfrak{A}_{[1,\infty)},$$

and

$$\varphi_h(Q) := \frac{\varphi\left(h^{\frac{1}{2}} Q h^{\frac{1}{2}}\right)}{\varphi(h)}, \quad Q \in \mathcal{B} \otimes \mathfrak{A}_{[1,\infty)}.$$

Then L_h gives a bounded operator on the Banach space F_θ and there exists $\delta_1 > 0$ and $C_1 > 0$ such that

$$|||L_h^n(Q) - \varphi_h(Q)||| \leq C_1 e^{-\delta_1 n} |||Q|||, \tag{3}$$

for all $n \in \mathbb{N}$ and $Q \in F_\theta$.

(iii) $\lim_{n\to\infty} \|L^n(1) - h\| = 0$.

(iv) *As a bounded operator on F_θ, L has a simple eigenvalue 1 and rest of the spectrum has modulus less than $e^{-\frac{\delta_1}{2}}$.*

Proof The proof is completely analogous to that in [M]. We omit the details. □

Now we consider a family of Ruelle transfer operators $\{L_\alpha\}_{\alpha\in\mathbb{R}}$.

Theorem 2.2 *Let $\{L_\alpha\}_{\alpha\in\mathbb{R}}$ be a family of operators on $\mathcal{B} \otimes \mathfrak{A}_{[1,\infty)}$. Suppose that each L_α is of the form (2) with $a = a(\alpha) \in \mathfrak{A}_{[1,\infty)}$ and $\mathcal{E} : \mathcal{B} \otimes M_d(\mathbb{C}) \to \mathcal{B}$, satisfying (i), (iii) of Assumption 2.1. Assume that the map*

$$\mathbb{R} \ni \alpha \mapsto L_\alpha \in B(F_\theta)$$

has a $B(F_\theta)$-valued analytic extension to a neighborhood of $\mathbb{R}$. Then, as a bounded operator on F_θ, each L_α has a strictly positive simple eigenvalue $\lambda(\alpha)$ such that

(i) *$\lambda(\alpha)$ has a strictly positive eigenvector $h(\alpha) \in F_\theta$, and*

$$\lim_{n\to\infty} \left\|\lambda(\alpha)^{-n} L_\alpha^n(1) - h(\alpha)\right\| = 0,$$

(ii) *$\mathbb{R} \ni \alpha \mapsto \lambda(\alpha)$ is differentiable.*

Remark 2.1 *An analogous result for left-side chain $\mathfrak{A}_{(-\infty,-1]}$ holds.*

Proof
There exists a state φ_α and a strictly positive scalar $\lambda(\alpha)$ such that $L_\alpha^* \varphi_\alpha = \lambda(\alpha)\varphi_\alpha$. In fact, by the invertibility of $a(\alpha)$ and unitality of $\mathcal{E}$, we have

$$L_\alpha(1) \geq \left\|a(\alpha)^{-1}\right\|^{-2} > 0.$$

Accordingly, if ν is a state of $\mathcal{B}\otimes\mathfrak{A}_{[1,\infty)}$, a state

$$G(\nu)(Q) := \frac{\nu(L_\alpha(Q))}{\nu(L_\alpha(1))}, \quad Q\in\mathcal{B}\otimes\mathfrak{A}_{[1,\infty)}$$

is well defined. This map G is weak*continuous on the state space. Therefore, using Schaudar Tychonov theorem, we can show the existence of a fixed point of G, i.e., a state φ_α and a strictly positive scalar $\lambda(\alpha)$ such that $L_\alpha^*\varphi_\alpha = \lambda(\alpha)\varphi_\alpha$. (See [A1]).

The operator $\lambda(\alpha)^{-1}L_\alpha$ satisfies Assumption 2.1. Applying Theorem 2.1 to $\lambda(\alpha)^{-1}L_\alpha$, we obtain (i). By (iv) of Theorem 2.1 and regular perturbation theory, differentiability of $\lambda(\alpha)$ can be proven. □

We will construct Ruelle operators L_α so that the eigenvalue $\lambda(\alpha)$ in Theorem 2.2 corresponds to the logarithmic moment generating function $f(\alpha)$ in (1).

3 Large deviation principle for KMS-states

Let Ψ be a finite range interaction and ω a unique (τ_Ψ,β)-KMS state. Let Φ be another finite range interaction. In this section, we prove large deviation principle of the distribution of $\frac{1}{n}H_\Phi([1,n])$ in ω, Theorem 1.1. By the Gärtner-Ellis Theorem, it suffices to show the existence and differentiability of the logarithmic moment generating function

$$f(\alpha) = \lim_{n\to\infty}\frac{1}{n}\log\omega(e^{\alpha H_\Phi([1,n])}), \quad \forall\alpha\in\mathbb{R}. \tag{4}$$

Lemma 3.1 *Let $p_n(\alpha)$ be*

$$p_n(\alpha) := Tr_{[1,n]}\left(e^{-\frac{\beta}{2}H_\Psi[1,n]}e^{\alpha H_\Phi[1,n]}e^{-\frac{\beta}{2}H_\Psi[1,n]}\right), \quad \alpha\in\mathbb{R}.$$

It suffices to prove the existence and differentiability of the limit

$$\lim_{n\to\infty}\frac{1}{n}\log p_n(\alpha), \quad \forall\alpha\in\mathbb{R}. \tag{5}$$

Proof In [LR], it was shown that there exists a positive constant C_1 such that

$$C_1^{-1}\omega_n \le \omega|_{\mathfrak{A}_{[1,n]}} \le C_1\omega_n, \tag{6}$$

where ω_n is a state on $\mathfrak{A}_{[1,n]}$ given by

$$\omega_n(A) = \frac{Tr_{[1,n]}e^{-\beta H_\Psi[1,n]}A}{Tr_{[1,n]}e^{-\beta H_\Psi[1,n]}}.$$

From this inequality, we have

$$\lim_{n\to\infty}\frac{1}{n}\left(\log p_n(\alpha) - \log\omega(e^{\alpha H_\Phi[1,n]}) - \log Tr_{[1,n]}e^{-\beta H_\Psi[1,n]}\right)$$
$$= \lim_{n\to\infty}\frac{1}{n}\left(\log\omega_n(e^{\alpha H_\Phi[1,n]}) - \log\omega(e^{\alpha H_\Phi[1,n]})\right) = 0.$$

As the existence of the limit

$$\lim_{n\to\infty}\frac{1}{n}\log Tr_{[1,n]}e^{-\beta H_\Psi[1,n]}$$

is known, it suffices to prove the existence and differentiability of the limit (5).□

By Lemma 3.1, we shall confine our attention to the analysis of $p_n(\alpha)$. We will freely use the notations in Appendix A.

We now define a family of Ruelle transfer operators $\{L_\alpha\}_{\alpha\in\mathbb{R}}$, given in the form of (2): we set $\mathcal{B}=M_d(\mathbb{C})$, and define a completely positive unital map $\mathcal{E}:M_d(\mathbb{C})\otimes M_d(\mathbb{C})\to M_d(\mathbb{C})$, through the formula $\mathcal{E}(a\otimes b):=d^{-1}Tr_{M_d(\mathbb{C})}(a)b$. Furthermore, for each $\alpha\in\mathbb{R}$, we define $a(\alpha)$ by

$$a(\alpha):=\tau_{\Phi_{[1,\infty)}}^{-i\frac{\alpha}{2}}\left(E_r\left(-\frac{\beta}{2}\hat{H}^r_\Psi(1);-\frac{\beta}{2}H_\Psi[2,\infty)\right)\right)E_r\left(\frac{\alpha}{2}\hat{H}^r_\Phi(1);\frac{\alpha}{2}H_\Phi[2,\infty)\right)\in\mathfrak{A}_{[1,\infty)}.$$

The Ruelle transfer operator on $\mathcal{B}\otimes\mathfrak{A}_{[1,\infty)}=\mathfrak{A}_{[0,\infty)}$ is given by

$$L_\alpha(Q):=\gamma_{-1}\left(d^{-1}Tr_{\{0\}}\otimes id_{[1,\infty)}\right)\left(a^*(\alpha)Qa(\alpha)\right). \tag{7}$$

In order to apply Theorem 2.2, we have to check that each L_α satisfies the Assumption 2.1, (i) and (iii) :

Lemma 3.2 *Each L_α, $\alpha\in\mathbb{R}$ satisfies the Assumption 2.1, (i),(iii).*

Proof It was shown in [A1] that for any local element Q in $\mathfrak{A}_{[1,\infty)}$, a subset $I\subset[1,\infty)$, and a finite range interaction Φ, $E_r(Q;H_\Phi(I))$ is an invertible element in $\mathfrak{A}_1$, which is the subalgebra of F_θ defined by (24). Furthermore, any element in $\mathfrak{A}_1$ is entire analytic for τ_{Φ_I}, and τ_{Φ_I} acts on $\mathfrak{A}_1$ as a group of automorphisms with one complex parameter.(See Appendix A.) Therefore, $a(\alpha)$ belongs to $\mathfrak{A}_1$ and invertible in $\mathfrak{A}_1$, so $a(\alpha)$ belongs to F_θ and invertible in F_θ. Hence, (i) of Assumption 2.1 is satisfied.

The proof of (iii) goes parallel to the argument in [M], where an example of Ruelle transfer operator was considered. We shall first write L^n_α in a more tractable form. By an inductive calculation, we obtain

$$L^n_\alpha(Q)=d^{-n}\gamma_{-n}\circ\left(Tr_{[0,n-1]}\otimes id_{[n,\infty)}\right)\circ\left(\tilde{a}^*_n(\alpha)Q\tilde{a}_n(\alpha)\right),$$

where we denoted $a(\alpha)\gamma_1(a(\alpha))\gamma_2(a(\alpha))\cdots\gamma_{(n-1)}(a(\alpha))$ by $\tilde{a}_n(\alpha)$. It is not hard to prove

$$\tilde{a}_n(\alpha)=\tau_{\Phi_{[1,\infty)}}^{-i\frac{\alpha}{2}}\left(E_r\left(-\frac{\beta}{2}\hat{H}^r_\Psi(n);-\frac{\beta}{2}H_\Psi([n+1,\infty)\right)\right)E_r\left(\frac{\alpha}{2}\hat{H}^r_\Phi(n);\frac{\alpha}{2}H_\Phi([n+1,\infty)\right),$$

using (25). Let $a_n(\alpha)$, $n\geq 2$ be

$$a_n(\alpha):=\tilde{a}_n(\alpha)e^{\frac{\beta}{2}H_\Psi[1,n-1]}e^{-\frac{\alpha}{2}H_\Phi[1,n-1]}.$$

Using the relation (25) again, we can show

$$
\begin{aligned}
a_n(\alpha) =& \tau^{-i\frac{\alpha}{2}}_{\Phi_{[1,\infty)}} \left(E_r \left(-\frac{\beta}{2} W^r_\Psi(n); -\frac{\beta}{2} \left(H_\Psi([1,n-1]) + H_\Psi[n+1,\infty) \right) \right) \right) \\
& E_r \left(\frac{\alpha}{2} W^r_\Phi(n); \frac{\alpha}{2} \left(H_\Phi[1,n-1] + H_\Phi([n+1,\infty)) \right) \right).
\end{aligned}
$$

Furthermore, we define a completely positive unital map $\varphi_n : \mathfrak{A}_{[0,\infty)} \to \mathfrak{A}_{[0,\infty)},\ n \geq 2$, by

$$
\begin{aligned}
\varphi_n(Q) :=& p^{-1}_{n-1}(\alpha) d^{-1} \gamma_{-n} \circ \left(Tr_{[0,n-1]} \otimes id_{[n,\infty)} \right) \\
& \left(e^{-\frac{\beta}{2} H_\Psi[1,n-1]} e^{\frac{\alpha}{2} H_\Phi[1,n-1]} Q e^{\frac{\alpha}{2} H_\Phi[1,n-1]} e^{-\frac{\beta}{2} H_\Psi[1,n-1]} \right).
\end{aligned}
$$

Using these notations, we can rewrite L^n_α as

$$
L^n_\alpha(Q) = d^{-(n-1)} p_{n-1}(\alpha) \varphi_n(a_n(\alpha)^* Q a_n(\alpha)), \quad n \geq 2. \tag{8}
$$

Next we evaluate (8), using the properties of $a_n(\alpha)$ given in Lemma A.1: that is,

$$
\lim_{n\to\infty} \|[Q, a_n(\alpha)]\| = 0, \quad \forall Q \in \mathfrak{A}_{loc}, \tag{9}
$$

and that there exists a positive constant C such that

$$
\sup_{n\in\mathbb{N}} \|a_n(\alpha)\|,\ \sup_{n\in\mathbb{N}} \left\| (a_n(\alpha))^{-1} \right\| < C. \tag{10}
$$

Let Q be any strictly positive element in $\mathfrak{A}_{[0,n_0]}$. By (9), we can choose $\varepsilon > 0$ and $N(Q) \in \mathbb{N}$ so that

$$
4C^3 \left\| Q^{\frac{1}{2}} \right\| \varepsilon \leq \inf Q,
$$

and

$$
N(Q) \geq n_0 + 1, \quad \left\| [Q^{\frac{1}{2}}, a_n(\alpha)] \right\| < \varepsilon,\ \forall n \geq N(Q).
$$

As φ_n is a completely positive unital map, we have $\|\varphi_n\| = \|\varphi_n(1)\| = 1$. Note that $\varphi_n(Q)$ is a scalar if $n - 1 \geq n_0$. Thus we get

$$
\begin{aligned}
& L^n_\alpha(Q) \leq d^{-(n-1)} p_{n-1}(\alpha) \left(C^2 \varphi_n(Q) + 2C \left\| Q^{\frac{1}{2}} \right\| \left\| [Q^{\frac{1}{2}}, a_n(\alpha)] \right\| \right) \\
& \leq d^{-(n-1)} p_{n-1}(\alpha) \left(\frac{1}{2C^2} + C^2 \right) \varphi_n(Q),
\end{aligned}
$$

and

$$
\begin{aligned}
& L^n_\alpha(Q) \geq d^{-(n-1)} p_{n-1}(\alpha) \left(-2C \left\| Q^{\frac{1}{2}} \right\| \left\| [Q^{\frac{1}{2}}, a_n(\alpha)] \right\| + \frac{1}{C^2} \varphi_n(Q) \right) \\
& \geq d^{-(n-1)} p_{n-1}(\alpha) \frac{1}{2C^2} \varphi_n(Q),
\end{aligned}
$$

for all $n \geq N(Q)$. Hence we obtain (iii) of Assumption 2.1:

$$L_\alpha^n(Q) \leq (1+2C^4) \inf L_\alpha^n(Q),$$

for all $n \geq N(Q)$. $\square$

Proof of Theorem 1.1
Note that

$$\mathbb{R} \ni \alpha \mapsto L_\alpha \in B(F_\theta)$$

has a $B(F_\theta)$-valued analytic extension to a neighborhood of $\mathbb{R}$. We thus can apply Theorem 2.2 to $\{L_\alpha\}$. Accordingly, each L_α has a strictly positive eigenvalue $\lambda(\alpha)$ associated with a strictly positive eigenvector $h(\alpha)$ such that

$$\lim_{n\to\infty} \left\| \lambda(\alpha)^{-n} L_\alpha^n(1) - h(\alpha) \right\| = 0.$$

Furthermore, $\mathbb{R} \ni \alpha \mapsto \lambda(\alpha)$ is differentiable. By (8) and (10), we have

$$d^{-(n-1)} p_{n-1}(\alpha) C^{-2} \leq L_\alpha^n(1) = d^{-(n-1)} p_{n-1}(\alpha) \varphi_n(a_n(\alpha)^* 1 a_n(\alpha)) \leq d^{-(n-1)} p_{n-1}(\alpha) C^2. \tag{11}$$

Hence for any state ν on $\mathfrak{A}_{[0,\infty)}$, we have

$$\begin{aligned} &\lim_{n\to\infty} \frac{1}{n-1} \left(\log p_{n-1}(\alpha) - \log \nu(\lambda(\alpha)^{-n} L_\alpha^n(1)) - n \log \lambda(\alpha) - (n-1) \log d \right) \\ &= \lim_{n\to\infty} \frac{1}{n} (\log p_n(\alpha)) - \log \lambda(\alpha) - \log d = 0. \end{aligned}$$

Therefore, the limit

$$\lim_{n\to\infty} \frac{1}{n} \log p_n(\alpha) = \log \lambda(\alpha) + \log d, \quad \forall \alpha \in \mathbb{R}. \tag{12}$$

exists and is differentiable. Applying Lemma 3.1, we have thus proved the Theorem. $\square$

4 Large deviation principle for C^*-finitely correlated states

In this section, we prove the large deviation principle for finitely correlated states, Theorem 1.2. Let ω be a C^*-finitely correlated state generated by a finite dimensional C^*-algebra $\mathcal{B}$, a completely positive unital map $\mathcal{E} : M_d(\mathbb{C}) \otimes \mathcal{B} \to \mathcal{B}$ and a faithful state ρ. By the translation invariance of ω, it suffices to show that the limit

$$\lim_{n\to\infty} \frac{1}{n} \log \omega \left(e^{\alpha H_\Phi[-n,-1]} \right) \tag{13}$$

exists and is differentiable. We define a completely positive unital map $\hat{\mathcal{E}}_1 : \mathcal{B} \to \mathcal{B}$ through the formula $\hat{\mathcal{E}}_1(b) := \mathcal{E}(1 \otimes b),\ b \in \mathcal{B}$.

Lemma 4.1 *It suffices to show the existence and differentiability of the limit (13) for ω generated by a triple $(\mathcal{B}, \mathcal{E}, \rho)$ satisfying the following condition: there exists a positive constant $s > 0$ such that*

$$s^{-1}\rho(b) \le \left(\hat{\mathcal{E}}_1\right)(b) \le s\rho(b), \quad 0 \le \forall b,\ b \in \mathcal{B}. \tag{14}$$

Proof It is known that every C^*-finitely correlated state has a unique decomposition as a *finite* convex combination of extremal periodic states, which are again C^*-finitely correlated [FNW]. That is, we can write ω as a finite sum $\omega = \sum_{i=1}^n \lambda_i \omega_i,\ 0 < \lambda_i,\ \sum_{i=1}^n \lambda_i = 1$, where each ω_i is an extremal p_i periodic state. Furthermore, ω_i is a C^*-finitely correlated state on $(M_d(\mathbb{C})^{\otimes p_i})_{\mathbb{Z}}$, generated by a triple $(\mathcal{B}_i, \mathcal{E}_i, \rho_i)$, such that 1 is the only eigenvector of $(\hat{\mathcal{E}}_i)_1$ with eigenvalue one, and rest of the spectrum has modulus strictly less than 1. Therefore, it suffices to consider ω generated by a completely positive map $\mathcal{E}$ such that $\hat{\mathcal{E}}_1$ has a simple eigenvalue 1 and rest of the spectrum has modulus strictly less than 1. We shall confine our attention to this case.

Next we claim that there exists an integer l and a positive constant $s > 0$ such that

$$s^{-1}\rho(b) \le \left(\hat{\mathcal{E}}_1\right)^l(b) \le s\rho(b), \quad 0 \le b,\ b \in \mathcal{B}. \tag{15}$$

To see this, let P be a spectral projection of $\hat{\mathcal{E}}_1$ corresponding to the eigenvalue 1, and set $\bar{P} = 1 - P$. By assumption, the range of P is $\mathbb{C}1$. As ρ is a faithful state on a finite dimensional C^*-algebra, there exists $c > 0$ such that $\hat{\rho} \ge c1$. Accordingly, we have $c\|b\| \le \rho(b),\ \forall b \ge 0, b \in \mathcal{B}$. By the assumption, if we take l large enough, we have

$$\left\|(\hat{\mathcal{E}}_1)^l \bar{P}(b)\right\| \le \frac{c}{2}\|b\|, \quad \forall b \in \mathcal{B}.$$

Furthermore, we have

$$\rho(b) = \lim_{n\to\infty} \rho\left(\hat{\mathcal{E}}_1^n(b)\right) = \rho(P(b)).$$

We thus obtain the claim: there exists l such that

$$\frac{1}{2}\rho(b) \le \rho(b) - \frac{c}{2}\|b\| \le \hat{\mathcal{E}}_1^l(b) = \hat{\mathcal{E}}_1^l(Pb) + \hat{\mathcal{E}}_1^l(\bar{P}b) = \rho(b) + \hat{\mathcal{E}}_1^l\left(\bar{P}(b)\right) \le \rho(b) + \frac{c}{2}\|b\| \le \frac{3}{2}\rho(b),$$

for $0 \le b,\ b \in \mathcal{B}$.

Note that ω is a C^*-finitely correlated state on $((M_d(\mathbb{C}))^{\otimes l})_{\mathbb{Z}}$, generated by $(\mathcal{B}, \mathcal{E}^{(l)}, \rho)$, where $\mathcal{E}^{(l)}$ is the l-th iterate of $\mathcal{E}$. Furthermore, we have $\widehat{\mathcal{E}^{(l)}}_1 = (\hat{\mathcal{E}}_1)^l$. Therefore, it suffices to consider ω generated by a triple $(\mathcal{B}, \mathcal{E}, \rho)$ satisfying (14). □

We shall confine our attention to ω satisfying (14).

As a transfer operator, we consider a map from $\mathfrak{A}_{(-\infty,-1]}\otimes\mathcal{B}$ to $\mathfrak{A}_{(-\infty,-1]}\otimes\mathcal{B}$. For each $\alpha\in\mathbb{R}$, we define L_α by

$$L_\alpha(Q) := \tau_{c-}\circ\left(id_{(-\infty,-2]}\otimes\mathcal{E}\right)\left(a^*(\alpha)Qa(\alpha)\right), \quad Q\in\mathfrak{A}_{(-\infty,-1]}\otimes\mathcal{B}$$

Here, $a(\alpha)$ is an element of $\mathfrak{A}_{(-\infty,-1]}$ given by

$$a(\alpha) := E_r\left(\frac{\alpha}{2}\hat{H}^l_\Phi(-1);\frac{\alpha}{2}H_\Phi(-\infty,-2]\right).$$

Lemma 4.2 *Each L_α, $\alpha\in\mathbb{R}$ satisfies (i), (iii) of Assumption 2.1.*

Proof As in Section 3, $a(\alpha)$ is an invertible element of F_θ and (i) holds. We prove (iii).We shall first write L_α^n in a more tractable form. By an inductive calculation, we obtain

$$L_\alpha^n(Q) = \left(\tau_{c-}\circ(id_{(-\infty,-2]}\otimes\mathcal{E})\right)^n\left(\tilde{a}_n(\alpha)^*Q\tilde{a}_n(\alpha)\right),$$

where

$$\tilde{a}_n(\alpha) := a(\alpha)\gamma_{-1}(a(\alpha))\cdots\gamma_{-(n-1)}(a(\alpha)).$$

Let $a_n(\alpha), n\geq 2$ be

$$a_n(\alpha) := \tilde{a}_n(\alpha)e^{-\frac{\alpha}{2}H_\Phi[-n+1,-1]}.$$

For each $n\geq 2$, we define a positive constant $p_n(\alpha)$, a completely positive map Φ_n by

$$\begin{aligned}
&p_n(\alpha) := \omega\left(e^{\alpha H_\Phi[-n+1,-1]}\right)\\
&\Phi_n(Q) := p_n^{-1}(\alpha)\left(\tau_{c-}\circ(id_{(-\infty,-2]}\otimes\mathcal{E})\right)^n\left(e^{\frac{\alpha}{2}H_\Phi[-n+1,-1]}Qe^{\frac{\alpha}{2}H_\Phi[-n+1,-1]}\right).
\end{aligned} \tag{16}$$

Using these notations, we can write L_α^n as

$$L_\alpha^n(Q) = p_n(\alpha)\Phi_n(a_n(\alpha)^*Qa_n(\alpha)), \quad Q\in\mathfrak{A}_{(-\infty,-1]}\otimes\mathcal{B}, \quad n\geq 2. \tag{17}$$

Next, note that for $R\in\mathfrak{A}_{[-n+1,-1]}\otimes\mathcal{B}$, $n\geq 2$, an element

$$\left(\tau_{c-}\circ(id_{(-\infty,-2]}\otimes\mathcal{E})\right)^{n-1}\left(e^{\frac{\alpha}{2}H_\Phi[-n+1,-1]}Re^{\frac{\alpha}{2}H_\Phi[-n+1,-1]}\right) \tag{18}$$

belongs to $1_{\mathfrak{A}_{(-\infty,-1]}}\otimes\mathcal{B}$, and (identifying $1_{\mathfrak{A}_{(-\infty,-1]}}\otimes\mathcal{B}$ with $\mathcal{B}$,)

$$\rho\left(\left(\tau_{c-}\circ(id_{(-\infty,-2]}\otimes\mathcal{E})\right)^{n-1}\left(e^{\alpha H_\Phi[-n+1,-1]}\right)\right) = \omega(e^{\alpha H_\Phi[-n+1,-1]}) = p_n(\alpha).$$

Accordingly,

$$\varphi_n(R) := p_n(\alpha)^{-1}\rho\left(\left(\tau_{c-}\circ(id_{(-\infty,-2]}\otimes\mathcal{E})\right)^{n-1}\left(e^{\frac{\alpha}{2}H_\Phi[-n+1,-1]}Re^{\frac{\alpha}{2}H_\Phi[-n+1,-1]}\right)\right)$$

defines a state on $\mathfrak{A}_{[-n+1,-1]} \otimes \mathcal{B}$. We claim

$$s^{-1}\varphi_n(R) \leq \Phi_n(R) \leq s\varphi_n(R), \quad \forall R \geq 0, \quad R \in \mathfrak{A}_{[-n+1,-1]} \otimes \mathcal{B}. \tag{19}$$

To see this, we denote (18) by $1_{\mathfrak{A}_{(-\infty,-1]}} \otimes b_R$. We have

$$\Phi_n(R) = p_n^{-1}(\alpha)\left(\tau_{c-} \circ (id_{(-\infty,-2]} \otimes \mathcal{E})\right)\left(\left(\tau_{c-} \circ (id_{(-\infty,-2]} \otimes \mathcal{E})\right)^{n-1}\left(e^{\frac{\alpha}{2}H_\Phi[-n+1,-1]}Re^{\frac{\alpha}{2}H_\Phi[-n+1,-1]}\right)\right)$$
$$= p_n^{-1}(\alpha)\left(1_{\mathfrak{A}_{(-\infty,-1]}} \otimes \hat{\mathcal{E}}_1(b_R)\right).$$

Therefore, from the bound (14), we obtain the claim:

$$s^{-1}\varphi_n(R) = s^{-1}p_n^{-1}(\alpha)\rho(b_R) \leq \Phi_n(R) = p_n^{-1}(\alpha)\left(1_{\mathfrak{A}_{(-\infty,-1]}} \otimes \hat{\mathcal{E}}_1(b_R)\right) \leq sp_n^{-1}(\alpha)\rho(b_R) = s\varphi_n(R). \tag{20}$$

From (19), we have $0 \leq \Phi_n(1) \leq s$. As Φ_n is completely positive, we obtain $\|\Phi_n\| = \|\Phi_n(1)\| \leq s$.

We now check the condition (iii). As in Section 3, there exists a positive constant $C > 0$ such that

$$\sup_{n\in\mathbb{N}} \|a_n(\alpha)\|, \quad \sup_{n\in\mathbb{N}} \left\|a_n(\alpha)^{-1}\right\| < C. \tag{21}$$

Furthermore, we have

$$\lim_{n\to\infty} \|[Q, a_n(\alpha)]\| = 0, \quad \forall Q \in \mathfrak{A}_{loc}.$$

For a strictly positive element Q in $\mathfrak{A}_{[-n_0,-1]} \otimes \mathcal{B}$, we can choose $\varepsilon > 0$ and $N(Q) \in \mathbb{N}$ so that

$$2\varepsilon \left\|Q^{\frac{1}{2}}\right\| C \leq \frac{1}{2C^2}s^{-2}\inf Q,$$

and

$$n_0 + 1 \leq N(Q), \quad \left\|[Q^{\frac{1}{2}}, a_n(\alpha)]\right\| < \varepsilon, \ \forall n \geq N(Q).$$

Thus, due to the inequality (19), for $n \geq N(Q)$, we have

$$L_\alpha^n(Q) = p_n(\alpha)\Phi_n(a_n(\alpha)^*Qa_n(\alpha)) \leq 2C\|\Phi_n\|\left\|[Q^{\frac{1}{2}}, a_n(\alpha)]\right\|\left\|Q^{\frac{1}{2}}\right\|p_n(\alpha) + C^2sp_n(\alpha)\varphi_n(Q)$$
$$\leq p_n(\alpha)\left(\frac{1}{2C^2}s^{-1} + C^2s\right)\varphi_n(Q),$$
$$L_\alpha^n(Q) \geq -2\|\Phi_n\|C\left\|[Q^{\frac{1}{2}}, a_n(\alpha)]\right\|\left\|Q^{\frac{1}{2}}\right\|p_n(\alpha) + \frac{1}{C^2}s^{-1}\varphi_n(Q)p_n(\alpha) \geq p_n(\alpha)\varphi_n(Q)\frac{1}{2C^2}s^{-1}.$$

Hence for $n \geq N(Q)$, we obtain

$$L_\alpha^n(Q) \leq 2C^2s\left(\frac{1}{2C^2}s^{-1} + C^2s\right)\inf L_\alpha^n(Q).$$

We thus showed (iii). □

Proof of Theorem 1.2
Note that the map

$$\mathbb{R} \ni \alpha \mapsto L_\alpha \in B(F_\theta)$$

has a $B(F_\theta)$-valued analytic extension to a neighborhood of $\mathbb{R}$. We thus can apply the left-side version of Theorem 2.2 to $\{L_\alpha\}$, and obtain

$$\lim_{n\to\infty} \left\| \lambda(\alpha)^{-n} L_\alpha^n(1) - h(\alpha) \right\| = 0,$$

for some strictly positive element $h(\alpha)$ in $\mathfrak{A}_{(-\infty,-1]} \otimes \mathcal{B}$ and a strictly positive constant $\lambda(\alpha)$. Furthermore, $\lambda(\alpha)$ is differentiable with respect to α. By (17), (19)and (21), we have

$$\frac{1}{sC^2} p_n(\alpha) \le C^{-2} p_n(\alpha)\Phi_n(1) \le L_\alpha^n(1) = p_n(\alpha)\Phi_n(a_n(\alpha)^* a_n(\alpha)) \le C^2 p_n(\alpha)\Phi_n(1) \le C^2 s p_n(\alpha). \tag{22}$$

For any state ν on $\mathfrak{A}_{(-\infty,-1]} \otimes \mathcal{B}$, we obtain

$$\lim_{n\to\infty} \frac{1}{n} \log \omega \left(e^{\alpha H_\Phi[-n,-1]} \right) = \lim_{n\to\infty} \frac{1}{n} \log p_n(\alpha) = \lim_{n\to\infty} \frac{1}{n} \log \nu \left({L_\alpha}^n(1) \right) = \log \lambda(\alpha). \tag{23}$$

As $\log \lambda(\alpha)$ is differentiable, we have proved the Theorem. □

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A Analyticity of local elements

Let I be any subset of $\mathbb{Z}$ and Φ a finite range interaction. We define a new interaction Φ_I by

$$\Phi_I(X) := \begin{cases} \Phi(X), & if\ X \subset I \\ 0, & otherwise \end{cases}.$$

This new interaction gives a time evolution τ_{Φ_I}. We define $\mathfrak{A}_1$ by

$$\mathfrak{A}_1 := \left\{ Q \in F_\theta \cap \mathfrak{A}_{[1,\infty)} : 0 < \theta < 1 \right\}. \tag{24}$$

In [A1], H.Araki showed that $\mathfrak{A}_1$ is a $*-$ algebra and that any element in $\mathfrak{A}_1$ is entire analytic for τ_{Φ_I}. For a local element Q, we define $E_r(Q; H_\Phi(I))$ by

$$E_r(Q; H_\Phi(I)) \equiv \sum_{n=0}^{\infty} \int_0^1 d\beta_1 \int_0^{\beta_1} d\beta_2 \cdots \int_0^{\beta_{n-1}} d\beta_n \tau_{\Phi_I}^{-i\beta_n}(Q) \cdots \tau_{\Phi_I}^{-i\beta_1}(Q).$$

It was shown in [A1] that $E_r(Q; H_\Phi(I))$ is an element in $\mathfrak{A}_1$. Furthermore, following relations hold:

$$\begin{aligned} E_r(Q_1+Q_2; H_\Phi(I)) &= E_r(Q_1; Q_2 + H_\Phi(I)) E_r(Q_2; H_\Phi(I)), \\ E_r(Q; H_\Phi(I)) \tau_{\Phi_I}^{-i}(Q') &= \tau_{\Phi_I+Q}^{-i}(Q') E_r(Q; H_\Phi(I)), \end{aligned} \tag{25}$$

for all $Q_1, Q_2, Q \in \mathfrak{A}_{loc}$ and $Q' \in \mathfrak{A}_1$. Here, τ_{Φ_I+Q} is a perturbed dynamics of τ_{Φ_I} by a bounded perturbation Q. If $Q \in \mathfrak{A}_{loc}$, then for any $x > 1$, there exists a constant C_x such that

$$\sup_{N\in\mathbb{N}} x^N \cdot \| E_r(Q; H_\Phi(I)) - E_r(Q; H_\Phi(I \cap [-N, +N])) \| \le C_x.$$

We use the following notations:

$$\begin{aligned} \hat{H}^r_\Phi(n) &:= \sum_{I\subset[1,\infty), I\cap[1,n]\neq\phi} \Phi(I) \quad \in \mathfrak{A}_{[1,\infty)} \cap \mathfrak{A}_{loc}, \\ \hat{H}^l_\Phi(n) &:= \sum_{I\subset(-\infty,-1], I\cap[-n,-1]\neq\phi} \Phi(I) \quad \in \mathfrak{A}_{(-\infty,-1]} \cap \mathfrak{A}_{loc}, \\ W^r_\Phi(n) &:= \sum_{I\subset[1,\infty), I\not\subset[1,n-1], I\not\subset[n+1,\infty)} \Phi(I) \quad \in \mathfrak{A}_{[1,\infty)} \cap \mathfrak{A}_{loc}, \\ W^l_\Phi(n) &:= \sum_{I\subset(-\infty,-1], I\not\subset[-n+1,-1], I\not\subset(-\infty,-n-1]} \Phi(I) \quad \in \mathfrak{A}_{(-\infty,-1]} \cap \mathfrak{A}_{loc}. \end{aligned}$$

We may apply the same argument as [A1] to show the following facts:

Lemma A.1 *Let Φ and Ψ be finite range interactions with range less than $r > 0$. Then operators*

$$\begin{aligned} a_n(\alpha) :=& \tau^{-i\frac{\alpha}{2}}_{\Phi_{[1,\infty)}} \left(E_r\left(-\frac{\beta}{2} W^r_\Psi(n); -\frac{\beta}{2}(H_\Psi[1,n] + H_\Psi[n+1,\infty)) \right) \right) \\ & \cdot E_r\left(\frac{\alpha}{2} W^r_\Phi(n); \frac{\alpha}{2}(H_\Phi[1,n] + H_\Phi[n+1,\infty)) \right), \\ a^N_n(\alpha) :=& \tau^{-i\frac{\alpha}{2}}_{\Phi_{[n-N,n+N]\cap[1,\infty)}} \left(E_r\left(-\frac{\beta}{2} W^r_\Psi(n); -\frac{\beta}{2}(H_\Psi([n-N,n]\cap[1,\infty)) + H_\Psi([n+1,n+N]) \right) \right) \\ & \cdot E_r\left(\frac{\alpha}{2} W^r_\Phi(n); \frac{\alpha}{2}(H_\Phi([n-N,n]\cap[1,\infty)) + H_\Phi[n+1,n+N] \right), \\ & \alpha, \beta \in \mathbb{C}, \quad n \in \mathbb{N} \end{aligned}$$

are well-defined invertible elements in $\mathfrak{A}_1$ and $\mathfrak{A}_{[n-N-r,n+N+r]\cap[1,\infty)}$, respectively. For any compact set S in $\mathbb{C}$, there exists a positive constant C_S such that

$$\begin{aligned} & \sup_{\alpha\in S} \sup_{n\in\mathbb{N}} \|a_n(\alpha)\|, \ \sup_{\alpha\in S} \sup_{n\in\mathbb{N}} \left\| (a_n(\alpha))^{-1} \right\| < C_S, \\ & \sup_{N\in\mathbb{N}} \sup_{\alpha\in S} \sup_{n\in\mathbb{N}} \left\| a^N_n(\alpha) \right\|, \ \sup_{N\in\mathbb{N}} \sup_{\alpha\in S} \sup_{n\in\mathbb{N}} \left\| \left(a^N_n(\alpha)\right)^{-1} \right\| < C_S. \end{aligned}$$

Furthermore, for any $x > 1$, there exists a positive constant C_x such that

$$\sup_{N\in\mathbb{N}} \sup_{\alpha\in S} \sup_{n\in\mathbb{N}} x^N \cdot \left\| a_n(\alpha) - a_n^N(\alpha) \right\| \leq C_x.$$

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