Constraints on the symmetry energy from neutron star equation of state

K. Miyazaki

E-mail: miyazakiro@rio.odn.ne.jp

Abstract

We develop an equation of state (EOS) for neutron star (NS) matter, which forbids the direct URCA cooling and satisfies the recent information on the mass and the radius, simultaneously. At sub-saturation densities, the symmetry energy of the EOS is well described by a function $E_{sym}(\rho) = 31.6\,(\rho/\rho_0)^{\gamma}$ with $0.70 \le \gamma \le 0.77$. This constraint on the density dependence of the symmetry energy is much severer than that obtained from the analysis of the isospin diffusion date in heavy-ion collisions. Consequently, we can obtain the valuable information on nuclear matter from the astrophysical observations of NSs.

1 Introduction

Thanks to the recent progress in the terrestrial experiments of heavy-ion reactions and the astronomical observations of neutron stars (NSs), there are renewed interests on the equation of state (EOS) of nuclear matter. Nevertheless, we cannot reach a consensus [1,2] on the stiffness of the EOS. The analysis [3] of K^+ production in heavy-ion reactions suggests a rather soft EOS, while the observations of massive NSs [4,5] strongly suggest stiff EOSs. On the other hand, there have been many efforts [6-15] to derive the incompressibility K, which is a well-known measure of the stiffness of EOS. At present, it is however fair to follow Ref. [15], which concludes that the analyses of the flow of nuclear matter in heavy-ion reactions are not inconsistent to the incompressibility in the range of 167MeV$\le K \le$ 380MeV.

We have to note that the incompressibility is concerned with the isoscalar part of the EOS while the symmetry energy is more important in asymmetric nuclear matter as the NS matter and the matter produced in heavy-ion reactions. In fact, the symmetry energy plays crucial roles [16,17] in determining the proton fraction and the radius of NS. Inversely, the reliable information on the proton fraction and the radius impose the constraint on the symmetry energy. According to the standard scenario of NS cooling, the direct URCA process is forbidden. As a result, the proton fraction is severely limited. On the other hand, the recent observations [18-20] strongly suggest the radii $R >$ 13km for the typical NSs of the gravitational masses $1.2M_\odot < M_G < 1.5M_\odot$. Moreover, it is widely expected [21-24] that the recently found mass-radius relation [25,26] of EXO

0748-676 is useful to constrain the EOS of NS matter. For an example, a famous EOS referred to APR [27] is ruled out.

In the next section we review the relativistic mean-field (RMF) model of symmetric nuclear matter developed in Ref. [28] and extend it to asymmetric nuclear matter. In section 3 we apply the model to NS matter and investigate the constraints on the EOS from the direct URCA cooling and the mass-radius relations of NSs. Then, the constraint on the symmetry energy is investigated in the comparison with the analyses [17,29,30] of the isospin diffusion data in heavy-ion collisions. Finally, we summarize our investigation in section 4.

2 The RMF model with modified vertices

2.1 Symmetric nuclear matter

Here, we review the RMF model in Ref. [28] because it is not familiar and is not in a free on-line journal. We first note that using the positive and negative energy projection operators for the Dirac nucleon with mass M,

$$\Lambda^{(\pm)}(p) = \frac{\pm \not{p} + M}{2M}, \tag{1}$$

the identity holds for a vertex function Γ:

$$\Gamma \equiv \Lambda^{(+)}(p')\,\Gamma\Lambda^{(+)}(p) + \Lambda^{(+)}(p')\,\Gamma\Lambda^{(-)}(p) + \Lambda^{(-)}(p')\,\Gamma\Lambda^{(+)}(p) + \Lambda^{(-)}(p')\,\Gamma\Lambda^{(-)}(p)\,. \tag{2}$$

However, Eq. (2) is not meaningful in nuclear matter, because a nucleon in the medium is not a physically observed particle in the positive-energy state but a quasi-particle and because the coupling constant is determined only for the former. We have no direct information on the vertex in the negative-energy state from experimental data. Therefore, there are several theoretical efforts [31-34] to derive $NN\gamma$ vertex in the complete space of energy.

In the present work, we are based on the Walecka σ-ω model [35] of nuclear matter. In contrast to the $NN\gamma$ vertex, the $NN\sigma$ and $NN\omega$ vertices are modified in a purely phenomenological way. We assume that the modified $NN\sigma$ vertex is

$$\begin{aligned} I \;\rightarrow\; & \left[\Lambda^{(+)}(p')\,I\Lambda^{(+)}(p) + \Lambda^{(-)}(p')\,I\Lambda^{(-)}(p)\right] \\ & + \lambda_\sigma \left[\Lambda^{(+)}(p')\,I\Lambda^{(-)}(p) + \Lambda^{(-)}(p')\,I\Lambda^{(+)}(p)\right], \\ = \;& \frac{(1-\lambda_\sigma)\,\not{p}'\not{p} + (1+\lambda_\sigma)\,M^2}{2M^2}, \end{aligned} \tag{3}$$

while the modified $NN\omega$ vertex is

$$\begin{aligned}\gamma^\mu &\to \lambda_\omega \left[\Lambda^{(+)}(p')\,\gamma^\mu \Lambda^{(+)}(p) + \Lambda^{(-)}(p')\,\gamma^\mu \Lambda^{(-)}(p)\right] \\ &\quad + \left[\Lambda^{(+)}(p')\,\gamma^\mu \Lambda^{(-)}(p) + \Lambda^{(-)}(p')\,\gamma^\mu \Lambda^{(+)}(p)\right], \\ &= \frac{(\lambda_\omega - 1)\,\not{p}'\gamma^\mu \not{p} + (\lambda_\omega + 1)\,M^2 \gamma^\mu}{2\,M^2}, \end{aligned} \tag{4}$$

where λ_σ and λ_ω are the phenomenological parameters to have values between 0 and 1. Consequently, in the mean-field approximation, the Lagrangian of the symmetric nuclear matter is

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\left(\vec{\not{p}} - M\right)\psi - \frac{1}{2}\,m_\sigma^2 \langle\sigma\rangle^2 + \frac{1}{2}\,m_\omega^2 \langle\omega_0\rangle^2 \\ &\quad + g_{NN\sigma}\langle\sigma\rangle \frac{(1-\lambda_\sigma)\left(\bar{\psi}\overleftarrow{\not{p}}\right)\left(\vec{\not{p}}\,\psi\right) + (1+\lambda_\sigma)\,M^2\,\bar{\psi}\psi}{2\,M^2} \\ &\quad - g_{NN\omega}\langle\omega_0\rangle \frac{(\lambda_\omega - 1)\left(\bar{\psi}\overleftarrow{\not{p}}\right)\gamma^0\left(\vec{\not{p}}\,\psi\right) + (\lambda_\omega + 1)\,M^2\,\bar{\psi}\,\gamma^0\psi}{2\,M^2}, \end{aligned} \tag{5}$$

where ψ is the field of nucleon, $\langle\sigma\rangle$ and $\langle\omega_0\rangle$ are the scalar and vector mean-fields, m_σ and m_ω are the masses of σ and ω mesons, and $g_{NN\sigma}$ and $g_{NN\omega}$ are their coupling constants. The resultant equation of nucleon is

$$\begin{aligned}&\Bigg\{\not{p} - M + \frac{g_{NN\sigma}\langle\sigma\rangle}{2\,M^2}\left[(1-\lambda_\sigma)\,p^2 + (1+\lambda_\sigma)\,M^2\right] \\ &\quad - \frac{g_{NN\omega}\langle\omega_0\rangle}{2\,M^2}\left[(\lambda_\omega - 1)\,\not{p}\,\gamma^0\not{p} + (\lambda_\omega + 1)\,M^2\gamma^0\right]\Bigg\}\,\psi = 0. \end{aligned} \tag{6}$$

We have to reduce Eq. (6) to the Dirac equation. In the RMF model with scalar and vector mean-fields [35], the equation of nucleon generally has a form:

$$\left(\not{p} - \gamma^0 V - M^*\right)\psi = 0, \tag{7}$$

where

$$M^* = M + S. \tag{8}$$

The S and V are the scalar and vector potentials of nucleon. Noting that we can multiply Eq. (7) by any factors, Eq. (6) is equated with the following equation:

$$\left(a_1\,\not{p}/M + b_1\gamma^0 + c_1\right)\left(b_2\gamma^0 + c_2\right)\left(\not{p} - \gamma^0 V - M^*\right)\psi = 0. \tag{9}$$

Therefore, we have

$$a_1 b_2 = \frac{1}{2}\,(1-\lambda_\omega)\,g_{NN\omega}\frac{\langle\omega_0\rangle}{M}, \tag{10}$$

$$a_1 c_2 = \frac{1}{2}\,(1-\lambda_\sigma)\,g_{NN\sigma}\frac{\langle\sigma\rangle}{M}, \tag{11}$$

$$a_1 b_2\, m^* + a_1 c_2\, v = 0, \tag{12}$$

$$b_1 c_2 + b_2\, c_1 = 0, \tag{13}$$

$$b_1 b_2 + c_1 c_2 - (a_1 c_2\, m^* + a_1 b_2\, v) = 1, \tag{14}$$

$$(b_1 c_2\, m^* + b_1 b_2\, v) + (b_2 c_1\, m^* + c_1 c_2\, v) = \frac{1}{2}\,(1+\lambda_\omega)\, g_{NN\omega} \frac{\langle \omega_0 \rangle}{M}, \tag{15}$$

$$(b_1 b_2\, m^* + b_1 c_2\, v) + (c_1 c_2\, m^* + b_2 c_1\, v) = 1 - \frac{1}{2}\,(1+\lambda_\sigma)\, g_{NN\sigma} \frac{\langle \sigma \rangle}{M}, \tag{16}$$

where $m^* \equiv M^*/M$ and $v \equiv V/M$. Utilizing Eqs. (10), (13) and (14), Eq. (15) becomes

$$v + (a_1 c_2\, m^* + a_1 b_2\, v)\, v = \frac{1}{2}\,(1+\lambda_\omega)\, g_{NN\omega} \frac{\langle \omega_0 \rangle}{M}. \tag{17}$$

Then, using Eq. (12), Eq. (17) becomes

$$v = \left((m^*)^2 - v^2 \right) a_1 b_2 + \frac{1}{2}\,(1+\lambda_\omega)\, g_{NN\omega} \frac{\langle \omega_0 \rangle}{M}. \tag{18}$$

Finally, substituting Eq. (10) again, we have

$$V = g^*_{NN\omega} \left\langle \omega_0 \right\rangle, \tag{19}$$

where

$$g^*_{NN\omega} = \frac{1}{2} \left[(1+\lambda_\omega) + (1-\lambda_\omega) \left((m^*)^2 - v^2 \right) \right] g_{NN\omega}. \tag{20}$$

On the other hand, utilizing Eqs. (11), (13) and (14), Eq. (16) becomes

$$m^* + (a_1 c_2\, m^* + a_1 b_2\, v)\, m^* = 1 - \frac{1}{2}\,(1-\lambda_\sigma)\, g_{NN\sigma} \frac{\langle \sigma \rangle}{M}. \tag{21}$$

Then, using Eq. (12), Eq. (21) becomes

$$1 - m^* = \left((m^*)^2 - v^2 \right) a_1 c_2 + \frac{1}{2}\,(1-\lambda_\sigma)\, g_{NN\sigma} \frac{\langle \sigma \rangle}{M}. \tag{22}$$

Finally, substituting Eq. (11) again, we have

$$M - M^* = -S = g^*_{NN\sigma} \left\langle \sigma \right\rangle, \tag{23}$$

where

$$g^*_{NN\sigma} = \frac{1}{2} \left[(1+\lambda_\sigma) + (1-\lambda_\sigma) \left((m^*)^2 - v^2 \right) \right] g_{NN\sigma}. \tag{24}$$

We can see that the modified vertices (3) and (4) have been reduced to Eqs. (20) and (24). They are the effective coupling constants of the quasi-particle in the nuclear matter, and so depend on the mean-fields. At zero density they should return to the free coupling constants. This is satisfied because of $\lim_{\rho\to 0} m^* = 1$ and $\lim_{\rho\to 0} v = 0$.

Consequently, we have the effective Lagrangian for symmetric nuclear matter, which is formally the same as the Walecka σ-ω model [35]:

$$\mathcal{L}_{eff} = \bar{\psi}\left(\not{p} - \gamma^0 V - M^*\right)\psi - \frac{1}{2} m_\sigma^2 \left\langle \sigma \right\rangle^2 + \frac{1}{2} m_\omega^2 \left\langle \omega_0 \right\rangle^2 . \tag{25}$$

The mean-fields $\langle\sigma\rangle$ and $\langle\omega_0\rangle$ are expressed by the effective mass m^* and the vector potential v through Eqs. (19) and (23). The energy per particle w is given in a unit of M by

$$w = \frac{1}{4}\left(3\,\frac{E_F^*}{M} + m^*\,\frac{\rho_S}{\rho}\right) + v - 1 + \frac{2}{G_\sigma \hat{\rho}}\left(\frac{1-m^*}{A_\sigma}\right)^2 - \frac{2}{G_\omega \hat{\rho}}\left(\frac{v}{A_\omega}\right)^2 , \tag{26}$$

where $\hat{\rho} \equiv \rho/\rho_0$ is the ratio of the baryon density to the saturation density, ρ_S is the scalar density and $E_F^* = \sqrt{k_F^2 + (M^*)^2}$ is the Fermi energy of nucleon. $A_{\sigma(\omega)}$ and $G_{\sigma(\omega)}$ are

$$A_{\sigma(\omega)} = \left(1 + \lambda_{\sigma(\omega)}\right) + \left(1 - \lambda_{\sigma(\omega)}\right)\left((m^*)^2 - v^2\right), \tag{27}$$

$$G_{\sigma(\omega)} = \frac{g_{NN\sigma(\omega)}^2 \rho_0}{m_{\sigma(\omega)}^2 M}. \tag{28}$$

The effective mass m^* and the vector potential v are determined by extremizing w, $\partial w/\partial m^* = 0$ and $\partial w/\partial v = 0$. From the equations we can express the coupling constants in terms of m^* and v:

$$G_\sigma = \frac{4\,C}{A_\sigma^3 B_\sigma \hat{\rho}}\left(1 - m^*\right), \tag{29}$$

$$G_\omega = \frac{4\,C}{A_\omega^3 B_\omega \hat{\rho}}\, v, \tag{30}$$

where

$$B_\sigma = \left[(1+\lambda_\omega) + (1-\lambda_\omega)\left((m^*)^2 + v^2\right)\right](\rho_S/\rho) + 2\,(1-\lambda_\omega)\, m^* v, \tag{31}$$

$$B_\omega = (\lambda_\sigma + 1) + (\lambda_\sigma - 1)\left((m^*)^2 - 2m^* + v^2\right) + 2\,(1-\lambda_\sigma)\,(1-m^*)\, v\,(\rho_S/\rho), \tag{32}$$

$$\begin{aligned} C \;=\; & \left[(\lambda_\sigma + 1) + (\lambda_\sigma - 1)\left((m^*)^2 - 2m^* + v^2\right)\right]\left[(1+\lambda_\omega) + (1-\lambda_\omega)\left((m^*)^2 + v^2\right)\right] \\ & - 4\,(1-\lambda_\sigma)\,(1-\lambda_\omega)\,(1-m^*)\, m^* v^2. \end{aligned} \tag{33}$$

(It is noted that Eqs. (41) and (42) in Ref. [28], which correspond to Eqs. (32) and (33) in the present work, were mistyped.)

2.2 Asymmetric nuclear matter

Here, we extend the model in the preceding subsection to asymmetric nuclear matter, for which the isovector scalar meson δ and vector meson ρ should be taken into account. Their modified vertices are introduced in analogous ways to σ and ω. The resultant equation of nucleon is

$$\begin{aligned}\Big\{ \not{p} - M &+ \frac{g_{NN\sigma}\left<\sigma\right>}{2\,M^2}\left[(1-\lambda_\sigma)\,p^2 + (1+\lambda_\sigma)\,M^2\right] \\ &+ \frac{g_{NN\delta}\left<\delta_3\right>}{2\,M^2}\left[(1-\lambda_\delta)\,p^2 + (1+\lambda_\delta)\,M^2\right]\tau_3 \\ &- \frac{g_{NN\omega}\left<\omega_0\right>}{2\,M^2}\left[(\lambda_\omega-1)\,\not{p}\,\gamma^0\not{p} + (\lambda_\omega+1)\,M^2\gamma^0\right] \\ &- \frac{g_{NN\rho}\left<\rho_{03}\right>}{2\,M^2}\left[(\lambda_\rho-1)\,\not{p}\,\gamma^0\not{p} + (\lambda_\rho+1)\,M^2\gamma^0\right]\tau_3\Big\}\,\psi = 0. \end{aligned} \tag{34}$$

Then, we consider the equations for proton and neutron independently. The procedure following Eq. (6) is readily applied to them. It is easily seen that the effective NNX ($X=\sigma$, ω, δ and ρ) coupling constant is given by

$$g^*_{pp(nn)X} = h_{pp(nn)X}\,g_{NNX} = \frac{1}{2}\left[(1+\lambda_X) + (1-\lambda_X)\left(\left(m^*_{p(n)}\right)^2 - v^2_{p(n)}\right)\right] g_{NNX}. \tag{35}$$

Consequently, the effective Lagrangian for asymmetric nuclear matter is

$$\begin{aligned}\mathcal{L}_{eff} &= \bar{\psi}_p\left(\not{p} - \gamma^0 V_p - M_p^*\right)\psi_p + \bar{\psi}_n\left(\not{p} - \gamma^0 V_n - M_n^*\right)\psi_n \\ &\quad - \frac{1}{2}\,m_\sigma^2\left<\sigma\right>^2 + \frac{1}{2}\,m_\omega^2\left<\omega_0\right>^2 - \frac{1}{2}\,m_\delta^2\left<\delta_3\right>^2 + \frac{1}{2}\,m_\rho^2\left<\rho_{03}\right>^2. \end{aligned} \tag{36}$$

where ψ_p and ψ_n are the Dirac fields of proton and neutron. The scalar and vector potentials are given by

$$S_p = -\,g^*_{pp\sigma}\left<\sigma\right> - g^*_{pp\delta}\left<\delta_3\right>, \tag{37}$$

$$S_n = -\,g^*_{nn\sigma}\left<\sigma\right> + g^*_{nn\delta}\left<\delta_3\right>, \tag{38}$$

$$V_p = g^*_{pp\omega}\left<\omega_0\right> + g^*_{pp\rho}\left<\rho_{03}\right>, \tag{39}$$

$$V_n = g^*_{nn\omega}\left<\omega_0\right> - g^*_{nn\rho}\left<\rho_{03}\right>. \tag{40}$$

The energy density of asymmetric nuclear matter is

$$\begin{aligned}\mathcal{E} &= \sum_{i=p,n}\left\{\frac{1}{4}\left(3E^*_{Fi}\rho_i + M_i^*\rho_{Si}\right) + V_i\,\rho_i\right\} \\ &\quad + \frac{1}{2}\,m_\sigma^2\left<\sigma\right>^2 + \frac{1}{2}\,m_\delta^2\left<\delta_3\right>^2 - \frac{1}{2}\,m_\omega^2\left<\omega_0\right>^2 - \frac{1}{2}\,m_\rho^2\left<\rho_{03}\right>^2, \end{aligned} \tag{41}$$

where ρ_i and ρ_{Si} are the baryon density and the scalar density of proton or neutron. From Eqs. (37)-(40), the meson mean-fields are expressed in terms of the effective masses

$M^*_{p(n)} = m^*_{p(n)} M$ and the vector potentials $V_{p(n)} = v_{p(n)} M$:

$$\langle \sigma \rangle = \frac{M}{g_{NN\sigma}} \frac{h_{nn\delta}\left(1 - m^*_p\right) + h_{pp\delta}\left(1 - m^*_n\right)}{H_S}, \tag{42}$$

$$\langle \delta_3 \rangle = \frac{M}{g_{NN\delta}} \frac{h_{nn\sigma}\left(1 - m^*_p\right) - h_{pp\sigma}\left(1 - m^*_n\right)}{H_S}, \tag{43}$$

$$\langle \omega_0 \rangle = \frac{M}{g_{NN\omega}} \frac{h_{nn\rho} v_p + h_{pp\rho} v_n}{H_V}, \tag{44}$$

$$\langle \rho_{03} \rangle = \frac{M}{g_{NN\rho}} \frac{h_{nn\omega} v_p - h_{pp\omega} v_n}{H_V}, \tag{45}$$

where

$$H_S = h_{pp\sigma} h_{nn\delta} + h_{nn\sigma} h_{pp\delta}, \tag{46}$$

$$H_V = h_{pp\omega} h_{nn\rho} + h_{nn\omega} h_{pp\rho}. \tag{47}$$

It is noted that the scalar mean-fields depend on the vector potentials.

The effective masses and the vector potentials are determined by extremizing $\mathcal{E}$:

$$\begin{aligned} \frac{\partial \mathcal{E}}{\partial V_{p(n)}} &= \rho_{p(n)} + m_\sigma^2 \frac{\langle \sigma \rangle}{M} \frac{\partial \langle \sigma \rangle}{\partial v_{p(n)}} + m_\delta^2 \frac{\langle \delta_3 \rangle}{M} \frac{\partial \langle \delta_3 \rangle}{\partial v_{p(n)}} \\ &\quad - m_\omega^2 \frac{\langle \omega_0 \rangle}{M} \frac{\partial \langle \omega_0 \rangle}{\partial v_{p(n)}} - m_\rho^2 \frac{\langle \rho_{03} \rangle}{M} \frac{\partial \langle \rho_{03} \rangle}{\partial v_{p(n)}} = 0, \end{aligned} \tag{48}$$

$$\begin{aligned} \frac{\partial \mathcal{E}}{\partial M^*_{p(n)}} &= \rho_{Sp(n)} + m_\sigma^2 \frac{\langle \sigma \rangle}{M} \frac{\partial \langle \sigma \rangle}{\partial m^*_{p(n)}} + m_\delta^2 \frac{\langle \delta_3 \rangle}{M} \frac{\partial \langle \delta_3 \rangle}{\partial m^*_{p(n)}} \\ &\quad - m_\omega^2 \frac{\langle \omega_0 \rangle}{M} \frac{\partial \langle \omega_0 \rangle}{\partial m^*_{p(n)}} - m_\rho^2 \frac{\langle \rho_{03} \rangle}{M} \frac{\partial \langle \rho_{03} \rangle}{\partial m^*_{p(n)}} = 0, \end{aligned} \tag{49}$$

where

$$\frac{\partial \langle \sigma \rangle}{\partial v_p} = \frac{v_p}{H_S} \left\{ \left[\left(1 - \lambda_\delta\right) h_{nn\sigma} + \left(1 - \lambda_\sigma\right) h_{nn\delta} \right] \langle \sigma \rangle - \left(1 - \lambda_\delta\right) \frac{\left(1 - m^*_n\right) M}{g_{NN\delta}} \right\}, \tag{50}$$

$$\frac{\partial \langle \delta_3 \rangle}{\partial v_p} = \frac{v_p}{H_S} \left\{ \left[\left(1 - \lambda_\delta\right) h_{nn\sigma} + \left(1 - \lambda_\sigma\right) h_{nn\delta} \right] \langle \delta_3 \rangle + \left(1 - \lambda_\sigma\right) \frac{\left(1 - m^*_n\right) M}{g_{NN\delta}} \right\}, \tag{51}$$

$$\frac{\partial \langle \omega_0 \rangle}{\partial v_p} = \frac{M}{g_{NN\omega}} \frac{h_{nn\rho}}{H_V} + \frac{v_p}{H_V} \left\{ \left[\left(1 - \lambda_\rho\right) h_{nn\omega} + \left(1 - \lambda_\omega\right) h_{nn\rho} \right] \langle \omega_0 \rangle - \left(1 - \lambda_\rho\right) \frac{v_n M}{g_{NN\omega}} \right\}, \tag{52}$$

$$\frac{\partial \langle \rho_{03} \rangle}{\partial v_p} = \frac{M}{g_{NN\rho}} \frac{h_{nn\omega}}{H_V} + \frac{v_p}{H_V} \left\{ \left[\left(1 - \lambda_\rho\right) h_{nn\omega} + \left(1 - \lambda_\omega\right) h_{nn\rho} \right] \langle \rho_{03} \rangle + \left(1 - \lambda_\omega\right) \frac{v_n M}{g_{NN\rho}} \right\}, \tag{53}$$

$$\frac{\partial\langle\sigma\rangle}{\partial v_n}=\frac{v_n}{H_S}\left\{\left[(1-\lambda_\delta)h_{pp\sigma}+(1-\lambda_\sigma)h_{pp\delta}\right]\langle\sigma\rangle-(1-\lambda_\delta)\frac{\left(1-m_p^*\right)M}{g_{NN\sigma}}\right\}, \tag{54}$$

$$\frac{\partial\langle\delta_3\rangle}{\partial v_n}=\frac{v_n}{H_S}\left\{\left[(1-\lambda_\delta)h_{pp\sigma}+(1-\lambda_\sigma)h_{pp\delta}\right]\langle\delta_3\rangle-(1-\lambda_\sigma)\frac{\left(1-m_p^*\right)M}{g_{NN\delta}}\right\}, \tag{55}$$

$$\frac{\partial\langle\omega_0\rangle}{\partial v_n}=\frac{M}{g_{NN\omega}}\frac{h_{pp\rho}}{H_V}+\frac{v_n}{H_V}\left\{\left[(1-\lambda_\rho)h_{pp\omega}+(1-\lambda_\omega)h_{pp\rho}\right]\langle\omega_0\rangle-(1-\lambda_\rho)\frac{v_pM}{g_{NN\omega}}\right\}, \tag{56}$$

$$\frac{\partial\langle\rho_{03}\rangle}{\partial v_n}=-\frac{M}{g_{NN\rho}}\frac{h_{pp\omega}}{H_V}+\frac{v_n}{H_V}\left\{\left[(1-\lambda_\rho)h_{pp\omega}+(1-\lambda_\omega)h_{pp\rho}\right]\langle\rho_{03}\rangle-(1-\lambda_\omega)\frac{v_pM}{g_{NN\rho}}\right\}, \tag{57}$$

$$\frac{\partial\langle\sigma\rangle}{\partial m_p^*}=-\frac{M}{g_{NN\sigma}}\frac{h_{nn\delta}}{H_S}+\frac{m_p^*}{H_S}\left\{(1-\lambda_\delta)\frac{\left(1-m_n^*\right)M}{g_{NN\sigma}}-\left[(1-\lambda_\delta)h_{nn\sigma}+(1-\lambda_\sigma)h_{nn\delta}\right]\langle\sigma\rangle\right\}, \tag{58}$$

$$\frac{\partial\langle\delta_3\rangle}{\partial m_p^*}=-\frac{M}{g_{NN\delta}}\frac{h_{nn\sigma}}{H_S}-\frac{m_p^*}{H_S}\left\{(1-\lambda_\sigma)\frac{\left(1-m_n^*\right)M}{g_{NN\delta}}+\left[(1-\lambda_\delta)h_{nn\sigma}+(1-\lambda_\sigma)h_{nn\delta}\right]\langle\delta_3\rangle\right\}, \tag{59}$$

$$\frac{\partial\langle\omega_0\rangle}{\partial m_p^*}=\frac{m_p^*}{H_V}\left\{(1-\lambda_\rho)\frac{v_nM}{g_{NN\omega}}-\left[(1-\lambda_\rho)h_{nn\omega}+(1-\lambda_\omega)h_{nn\rho}\right]\langle\omega_0\rangle\right\}, \tag{60}$$

$$\frac{\partial\langle\rho_{03}\rangle}{\partial m_p^*}=-\frac{m_p^*}{H_V}\left\{(1-\lambda_\omega)\frac{v_nM}{g_{NN\rho}}+\left[(1-\lambda_\rho)h_{nn\omega}+(1-\lambda_\omega)h_{nn\rho}\right]\langle\rho_{03}\rangle\right\}, \tag{61}$$

$$\frac{\partial\langle\sigma\rangle}{\partial m_n^*}=-\frac{M}{g_{NN\sigma}}\frac{h_{pp\delta}}{H_S}+\frac{m_n^*}{H_S}\left\{(1-\lambda_\delta)\frac{\left(1-m_p^*\right)M}{g_{NN\sigma}}-\left[(1-\lambda_\delta)h_{pp\sigma}+(1-\lambda_\sigma)h_{pp\delta}\right]\langle\sigma\rangle\right\}, \tag{62}$$

$$\frac{\partial\langle\delta_3\rangle}{\partial m_n^*}=\frac{M}{g_{NN\delta}}\frac{h_{pp\sigma}}{H_S}+\frac{m_n^*}{H_S}\left\{(1-\lambda_\sigma)\frac{\left(1-m_p^*\right)M}{g_{NN\delta}}-\left[(1-\lambda_\delta)h_{pp\sigma}+(1-\lambda_\sigma)h_{pp\delta}\right]\langle\delta_3\rangle\right\}, \tag{63}$$

$$\frac{\partial\langle\omega_0\rangle}{\partial m_n^*}=\frac{m_n^*}{H_V}\left\{(1-\lambda_\rho)\frac{v_pM}{g_{NN\omega}}-\left[(1-\lambda_\rho)h_{pp\omega}+(1-\lambda_\omega)h_{pp\rho}\right]\langle\omega_0\rangle\right\}, \tag{64}$$

$$\frac{\partial\langle\rho_{03}\rangle}{\partial m_n^*}=\frac{m_n^*}{H_V}\left\{(1-\lambda_\omega)\frac{v_pM}{g_{NN\rho}}-\left[(1-\lambda_\rho)h_{pp\omega}+(1-\lambda_\omega)h_{pp\rho}\right]\langle\rho_{03}\rangle\right\}. \tag{65}$$

3 Numerical analyses

We first determine the isoscalar vertices. For given values of λ_σ and λ_ω, $g_{NN\sigma}$ and $g_{NN\omega}$ are determined so as to reproduce the nuclear matter saturation:

$$\partial w/\partial\rho\,|_{\hat\rho=1}=0, \tag{66}$$

$$w_0M\equiv w\left(\hat\rho=1\right)M=-15.75\text{MeV}. \tag{67}$$

Substituting Eqs. (29) and (30) into Eqs. (66) and (67), we have a self-consistency equation for m^* at saturation:

$$\frac{1}{4}\left(\frac{E_F^*}{M} - m_0^* \frac{\rho_{S0}}{\rho_0}\right) - \frac{A_{\sigma 0}B_{\sigma 0}}{2\,C_0}\,(1-m_0^*) + \frac{A_{\omega 0}B_{\omega 0}}{2\,C_0}\,v_0 = 0, \tag{68}$$

where

$$v_0 = 1 + w_0 - E_F^*/M. \tag{69}$$

(Indices 0 mean that the values are evaluated at saturation density $\rho = \rho_0 = 0.16\text{fm}^{-3}$.) Solving the nonlinear equation (68) for the given λ_σ and λ_ω, we obtain the value of m_0^* and then v_0 from Eq. (69). The coupling constants $g_{NN\sigma}$ and $g_{NN\omega}$ are determined again from Eqs. (29) and (30). It is still necessary to fix the values of λ_σ and λ_ω. In this work, we simply assume $\lambda_\sigma = \lambda_\omega = \lambda_0$. Then we have a value $\lambda_0 = 2/3$ so as to reproduce $m_0^* \simeq 0.6$, which is required [36] for the reasonable spin-orbit splitting of finite nuclei. Table 1 summarizes the properties of the saturated nuclear matter. Although the incompressibility is relatively high, the value is tolerable as shown in Ref. [15].

Next, the isovector vertices should be determined. We simply assume $\lambda_\delta = \lambda_\rho = \lambda_1$ again and investigate the three cases of $\lambda_1 = 0.0$, 0.2 and 0.4 in the following. The $g_{NN\delta}$ is assumed to be the same as the Bonn A potential in Ref. [38], while the $g_{NN\rho}$ is adjusted so as to reproduce the empirical symmetry energy $E_{sym}\,(\rho_0) = 31.6\text{MeV}$ [29,30]. The obtained values are $g_{NN\rho}^2/(4\pi) = 2.98$, 2.44 and 2.05 for $\lambda_1 = 0.0$, 0.2 and 0.4, respectively. They are much larger than the Bonn A potential. This is a problem common to all the RMF models.

Then, we develop the EOS of NS matter. For the purpose we have to add the contributions of electron and muon to the Lagrangian (36). Because the leptons are treated as the free Fermi gases, the energy density (41) has an additional term $(1/4)\sum_{l=e^-,\mu^-}\left(3E_{Fl}\rho_l + m_l\rho_{Sl}\right)$. Although Eqs. (48) and (49) are not varied, the asymmetry is prescribed by the β-equilibrium condition

$$\mu_n - \mu_p = \mu_{e^-} = \mu_{\mu^-}. \tag{70}$$

The two independent bario-chemical potential μ_n and the charge chemical potential μ_{e^-} are determined through the baryon number conservation

$$\rho = \rho_p + \rho_n, \tag{71}$$

and the charge neutral condition

$$\rho_p = \rho_{e^-} + \rho_{\mu^-}. \tag{72}$$

For a given density ρ Eqs. (48), (49), (71) and (72) are solved numerically using the 6-rank Newton-Raphson method so that we have the values of $m_{p(n)}^*$, $v_{p(n)}$, μ_n and μ_{e^-}. For $\lambda_1 = 0.0$ we have no solution above $\rho = 1.10\text{fm}^{-3}$. This is however not a problem because the central density of the most massive NS is lower than $\rho = 1.0\text{fm}^{-3}$ as will be shown below. The pressure of NS matter is calculated from the Gibbs-Duhem relation $P = \mu_n \rho - \mathcal{E}$. The resultant EOSs for $\lambda_1 = 0.0$ and 0.2 are tabulated in Table 2.

The black curves in Fig. 1 show the proton fractions in NS. As λ_1 is larger, the proton is more abundant. The direct URCA cooling is allowed if the fraction exceeds

$$f_p^{DU} = \frac{1}{1 + \left(1 + k_{Fe}/k_{Fp}\right)^3}, \tag{73}$$

where k_{Fe} and k_{Fp} are the Fermi momenta of electron and proton. The red curve shows f_p^{DU} for $\lambda_1 = 0.2$, while the corresponding results for $\lambda_1 = 0.0$ and 0.4 are only slightly lower and higher than the red curve and so are not shown in the figure. According to the standard scenario of NS cooling, the direct URCA process should be forbidden. Consequently, only $0.0 \leq \lambda_1 \leq 0.2$ is allowed.

The black curves in Fig. 2 are the mass-radius relations of NSs calculated by integrating the Tolman-Oppenheimer-Volkov equation [39]. For the crust of NS at low densities $\rho < 0.08\text{fm}^{-3}$, we use the EOSs by Feynman-Metropolis-Teller, Baym-Pethick-Sutherland and Negele-Vautherin from Ref. [40]. The gravitational masses of the most massive NSs are $M_G = 1.93M_\odot$, $1.94M_\odot$ and $1.95M_\odot$ for $\lambda_1 = 0.0$, 0.2 and 0.4, respectively. Their radii are $R = 11.6$km, 11.8km and 11.9km, respectively. Their central baryon densities are $\rho = 0.97\text{fm}^{-3}$, 0.95fm^{-3} and 0.93fm^{-3}, respectively. The red line is the mass-radius relation of EXO 0748-676 [26], while the horizontal blue line indicates the lower limit of the observed mass [5] of PSR J0751+1807. Our result satisfies both the constraints irrespective of the value of λ_1. If the massive NS with $M_G > 2M_\odot$ [4] would be confirmed, our model was ruled out. However, the masses of NSs other than the typical ones with $M_G < 1.5M_\odot$ are not well confirmed. In fact, Ref. [41] suggests that EXO 0748-676 is a typical NS with $M_G = 1.35M_\odot$ in contrast to the result of Ref. [26].

According to the constraint on the proton fraction in Fig. 1, the NS radius is severely restricted to the values in the solid and dotted curves. On the other hand, there are notable observations [18-20] suggesting that the radius of the typical NS with $1.2M_\odot < M_G < 1.5M_\odot$ is possibly larger than 13km. If they would be true, the famous EOS referred to APR [27] was ruled out, the possibilities of strange star and the quark matter core in NS [21] were also ruled out, and only the value $\lambda_1 = 0.2$ was accepted in our model as shown by the vertical green line. In this respect, it is recently expected [42-45] that the moderately accurate measurement of the moment-of-inertia of J0737-3039A in future will impose a significant constraint on the EOS of NS matter. For completeness we have calculated it in a slow-rotation approximation [42,45]. The result is $I_{45} = 1.53$, 1.57 and 1.61 for $\lambda_1 = 0.0$, 0.2 and 0.4, respectively.

At sub-saturation densities the symmetry energy [17,29,30,46] is well described by a function $E_{sym}(\rho) = 31.6\,(\rho/\rho_0)^\gamma$. The crosses in Fig. 3 are the symmetry energies in our EOS at $\rho = 0.02\text{fm}^{-3}$, 0.04fm^{-3}, $\cdots$, 0.16fm^{-3}. In fact, they are well approximated by $31.6\,(\rho/\rho_0)^{0.70}$, $31.6\,(\rho/\rho_0)^{0.77}$ and $31.6\,(\rho/\rho_0)^{0.84}$ shown by the dotted curves. According to the result of Fig. 1, we have a constraint $0.70 \leq \gamma \leq 0.77$ on the density dependence of the symmetry energy. On the other hand, the constraint from the analyses [17,29,30] of the isospin diffusion data in heavy-ion collisions is $0.69 \leq \gamma \leq 1.05$. Both the lower limits agree well with each other while our upper limit is much severer. If the radius of the typical NS would be larger than 13km, we had a unique value $\gamma = 0.77$ according to the result of Fig. 2.

4 Summary

We develop an EOS of NS matter based on the RMF model with the modified meson-nucleon vertices. They are reduced to the field-dependent effective coupling constants of quasi-particles in the medium. Historically, the field-dependent coupling was first proposed in the Zimanyi-Moszkowski (ZM) model [47]. Moreover, the present author developed the extended ZM model in Refs. [48-50]. Other extensions were also proposed in Refs. [51,52]. The model in this work has an advantage over the other two models because the model EZM2 [50] allows the direct URCA cooling and the model D3C [52] has a lot of parameters.

The constant λ_0 to specify the modified isoscalar vertices is fixed so that the effective nucleon mass in saturated nuclear matter has a physically reasonable value $M^* \simeq 0.6M$. The isoscalar coupling constants are determined so as to reproduce the saturation of the symmetric nuclear matter. On the other hand, the constant λ_1 to specify the modified isovector vertices is treated as a parameter. The $NN\rho$ coupling constant is determined so as to reproduce the symmetry energy at the saturation density.

We first investigate the proton fraction in NS matter. It is more abundant as λ_1 is larger. The standard scenario of NS cooling limits λ_1 to a value between 0.0 and 0.2. We next investigate the mass-radius relation of NS. The radius is larger as λ_1 is larger. If the radius of the typical NS would be larger than 13km as suggested in the recent observations, the value of λ_1 was fixed to 0.2. The possible massive NS PSR J0751+1807 and the mass-radius relation of EXO 0748-676 are reproduced irrespective of the value of λ_1.

Then, we calculate the symmetry energy below the saturation density. It is well described by a function $E_{sym}(\rho) = 31.6\,(\rho/\rho_0)^\gamma$. According to the constraint on the proton fraction, the value of γ is limited to $0.70 \leq \gamma \leq 0.77$. The limitation is much severer than $0.69 \leq \gamma \leq 1.05$ obtained from the analyses of the isospin diffusion data in heavy-ion collisions. The result indicates that we can obtain the valuable information on nuclear matter from astrophysical observations of NSs. We also note that both the lower limits of γ agree well with each other. It suggests that the physically reasonable value of γ is near 0.7. This conjecture is consistent to the recent analyses of the fragment isotopic yields in heavy-ion reactions [46] and the symmetry potential in nucleon-nucleus scattering [53]. In this respect, it is valuable to investigate the neutron skin thickness of heavy nuclei [54] in our RMF model. It is a subject of a future work.

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Table 1: The values of the $NN\sigma$ and $NN\omega$ coupling constants, the effective mass at saturation, the scalar and vector potentials at saturation, the incompressibility and the Coulomb coefficient [37].

$\frac{g_{NN\sigma}^2}{4\pi}$	$\frac{g_{NN\omega}^2}{4\pi}$	m^*	S (MeV)	V (MeV)	K (MeV)	K_{Coul} (MeV)
10.4	15.7	0.597	−379	304	320	−5.91

Table 2: The EOSs, the pressure P vs. the baryon density ρ and the energy density $\mathcal{E}$, for the core region $\rho \geq 0.08\text{fm}^{-3}$ in NS.

	$\lambda_1 = 0.0$		$\lambda_1 = 0.2$	
$\rho_B(\text{cm}^{-3})$	$\mathcal{E}(\text{g}\cdot\text{cm}^{-3})$	$P(\text{dyn}\cdot\text{cm}^{-2})$	$\mathcal{E}(\text{g}\cdot\text{cm}^{-3})$	$P(\text{dyn}\cdot\text{cm}^{-2})$
8.00E+37	1.350949E+14	5.717765E+32	1.349457E+14	6.359135E+32
1.00E+38	1.690862E+14	1.076133E+33	1.689302E+14	1.246090E+33
1.20E+38	2.032231E+14	1.911122E+33	2.030878E+14	2.220682E+33
1.40E+38	2.375510E+14	3.155382E+33	2.374652E+14	3.634276E+33
1.60E+38	2.721126E+14	4.885360E+33	2.721036E+14	5.533926E+33
1.80E+38	3.069539E+14	7.206874E+33	3.070452E+14	8.023045E+33
2.00E+38	3.421220E+14	1.019774E+34	3.423340E+14	1.117493E+34
2.20E+38	3.776622E+14	1.391685E+34	3.780124E+14	1.504450E+34
2.40E+38	4.136166E+14	1.840373E+34	4.141195E+14	1.966860E+34
2.60E+38	4.500230E+14	2.367832E+34	4.506907E+14	2.506577E+34
2.80E+38	4.869142E+14	2.974214E+34	4.877567E+14	3.123706E+34
3.00E+38	5.243180E+14	3.658042E+34	5.253431E+14	3.816791E+34
3.20E+38	5.622567E+14	4.416526E+34	5.634709E+14	4.583084E+34
3.40E+38	6.007476E+14	5.245897E+34	6.021557E+14	5.418868E+34
3.60E+38	6.398032E+14	6.141752E+34	6.414091E+14	6.319772E+34
3.80E+38	6.794319E+14	7.099344E+34	6.812382E+14	7.281060E+34
4.00E+38	7.196382E+14	8.113831E+34	7.216467E+14	8.297875E+34
4.20E+38	7.604236E+14	9.180454E+34	7.626352E+14	9.365425E+34
4.40E+38	8.017869E+14	1.029467E+35	8.042018E+14	1.047911E+35
4.60E+38	8.437249E+14	1.145223E+35	8.463425E+14	1.163461E+35
4.80E+38	8.862327E+14	1.264921E+35	8.890514E+14	1.282793E+35
5.00E+38	9.293038E+14	1.388205E+35	9.323217E+14	1.405543E+35
5.20E+38	9.729311E+14	1.514753E+35	9.761454E+14	1.531380E+35
5.40E+38	1.017106E+15	1.644276E+35	1.020514E+15	1.660007E+35
5.60E+38	1.061821E+15	1.776516E+35	1.065417E+15	1.791158E+35
5.80E+38	1.107067E+15	1.911243E+35	1.110847E+15	1.924595E+35
6.00E+38	1.152833E+15	2.048254E+35	1.156793E+15	2.060108E+35
6.20E+38	1.199112E+15	2.187367E+35	1.203244E+15	2.197507E+35
6.40E+38	1.245894E+15	2.328422E+35	1.250192E+15	2.336627E+35
6.60E+38	1.293170E+15	2.471276E+35	1.297627E+15	2.477317E+35

(continued from the preceding page)

6.80E+38	1.340930E+15	2.615804E+35	1.345539E+15	2.619445E+35
7.00E+38	1.389165E+15	2.761892E+35	1.393918E+15	2.762895E+35
7.20E+38	1.437868E+15	2.909441E+35	1.442755E+15	2.907560E+35
7.40E+38	1.487029E+15	3.058363E+35	1.492040E+15	3.053347E+35
7.60E+38	1.536639E+15	3.208578E+35	1.541766E+15	3.200172E+35
7.80E+38	1.586691E+15	3.360016E+35	1.591924E+15	3.347961E+35
8.00E+38	1.637177E+15	3.512613E+35	1.642504E+15	3.496645E+35
8.20E+38	1.688089E+15	3.666315E+35	1.693499E+15	3.646165E+35
8.40E+38	1.739420E+15	3.821070E+35	1.744901E+15	3.796466E+35
8.60E+38	1.791161E+15	3.976833E+35	1.796702E+15	3.947500E+35
8.80E+38	1.843308E+15	4.133565E+35	1.848894E+15	4.099221E+35
9.00E+38	1.895851E+15	4.291228E+35	1.901472E+15	4.251591E+35
9.20E+38	1.948786E+15	4.449790E+35	1.954427E+15	4.404574E+35
9.40E+38	2.002106E+15	4.609223E+35	2.007752E+15	4.558135E+35
9.60E+38	2.055803E+15	4.769500E+35	2.061442E+15	4.712246E+35
9.80E+38	2.109874E+15	4.930597E+35	2.115490E+15	4.866880E+35
1.00E+39	2.164311E+15	5.092493E+35	2.169889E+15	5.022011E+35
1.02E+39	2.219109E+15	5.255171E+35	2.224634E+15	5.177618E+35
1.04E+39	2.274263E+15	5.418611E+35	2.279719E+15	5.333680E+35
1.06E+39	2.329767E+15	5.582800E+35	2.335139E+15	5.490177E+35
1.08E+39	2.385617E+15	5.747725E+35	2.390888E+15	5.647093E+35
1.10E+39	2.441808E+15	5.913372E+35	2.446960E+15	5.804411E+35

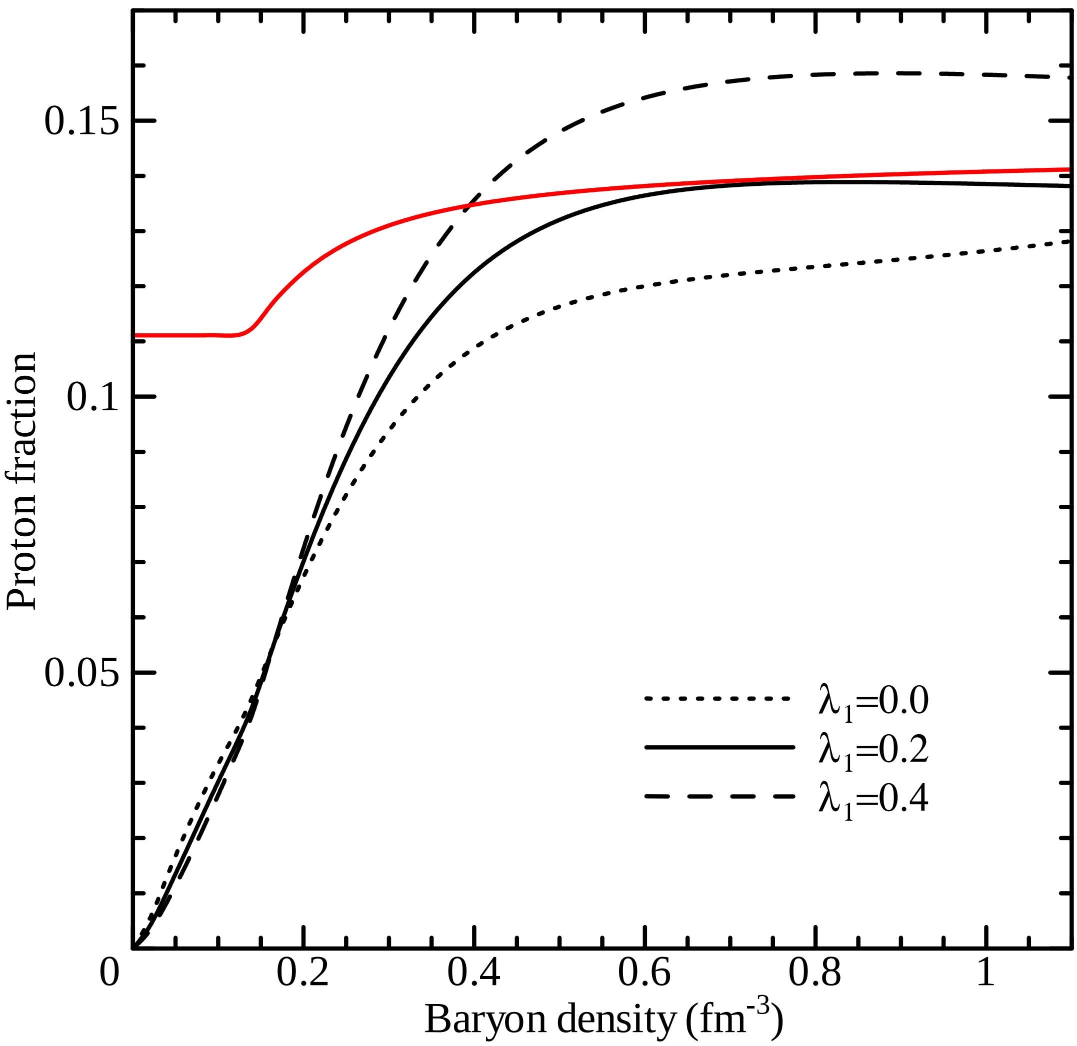

Figure 1: The black curves are the proton fractions for $\lambda_1 = 0.0$, 0.2 and 0.4 as functions of the baryon density. The red curve is the direct URCA limit (73) for $\lambda_1 = 0.2$.

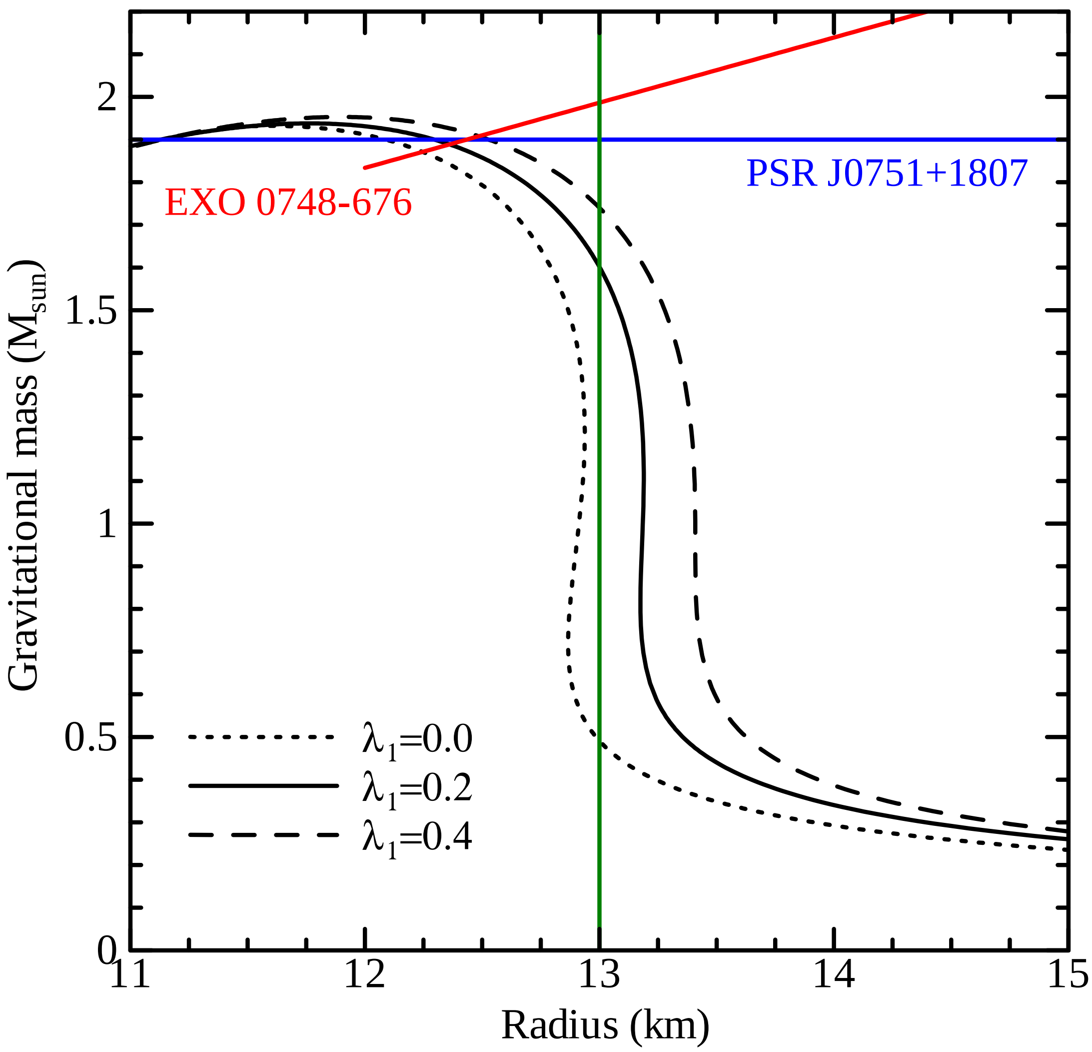

Figure 2: The black curves are the mass-radius relations of NSs for $\lambda_1 = 0.0$, 0.2 and 0.4. The red line is the mass-radius relation of EXO 0748-676 derived in Ref. [26]. The horizontal blue line indicates the lower limit of the observed gravitational mass [5] of PSR J0751+1807.

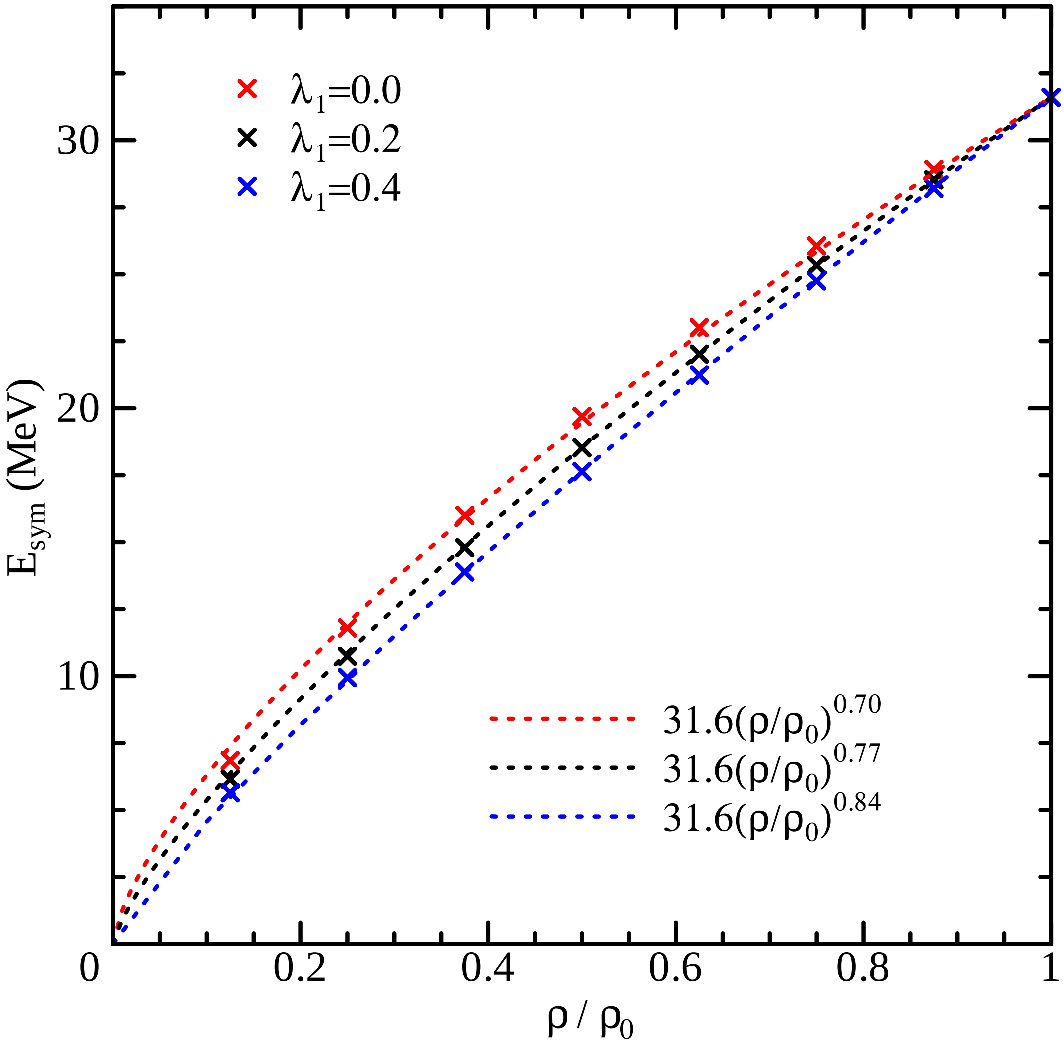

Figure 3: The crosses are the symmetry energies calculated at the densities from $\rho = 0.02\text{fm}^{-3}$ to $\rho_0 = 0.16\text{fm}^{-3}$ for $\lambda_1 = 0.0$, 0.2 and 0.4. They are interpolated by the dotted curves.