

Can newly born neutron star collapse to low-mass black hole?

K. Miyazaki

E-mail: miyazakiro@rio.odn.ne.jp

Abstract

We investigate for the first time the newly born neutron star (NS) containing both hyperons and kaons as strange hadrons within the relativistic mean-field theory. It is found that the maximum baryonic mass of the newly born NS is lower than that of the cold deleptonized NS. Against the suggestion by Brown and Bethe, there is no possibility of the delayed collapse of NS to low-mass black hole.

After Brown and Bethe [1] suggested low-mass black holes (BHs) from the delayed collapses of neutron stars (NSs), there have been several investigations [2-6] for the metastability of newly born NS. The NS is in quasi-stationary β -equilibrium state at different times during its evolution because the time scale of weak interaction is much shorter than the time scale of neutrino diffusion. The static approach adopted in Refs. [2-6] is therefore useful to newly born NSs. Then, it is expected that the baryon number or the baryonic mass is conserved in NSs. A newly born NS, whose baryonic mass is larger than the maximum baryonic mass of cold deleptonized NS, is therefore metastable and so collapses to a BH after deleptonization. In Refs. [2-6] there appear low-mass BHs whose gravitational masses are lower than two times of the solar mass. To the contrary, the observed low-mass BH candidates [7,8] have the masses near $4M_{\odot}$. Consequently, the possibility of low-mass BHs is still open to question.

The suggestion by Brown and Bethe was based on the phase transition in NS matter due to antikaon condensation, while Refs. [2-6] did not take into account it. So far, only two works [9,10] investigated the antikaon condensation in hot protonneutron star (PNS). Their results also supported the delayed collapses of NSs. However, both the works considered only nucleons as baryons but not hyperons. This is insufficient because the hyperons have strangeness as well as kaons. On the other hand, there are several works [11-16] for the cold deleptonized NS consisting of antikaons and all the baryons. In their results the antikaon condensations lead to noticeable soft equations-of-state (EOSs) for NS matter. Those EOSs would be much stiffer in hot PNS matter because of the thermal contribution to pressure, and so the maximum baryonic mass of newly born NS would be larger than that of the cold deleptonized NS. Consequently, the delayed collapses of NSs

to low-mass BHs would be still possible even if the models of Refs. [11-16] were applied to newly born NS.

To the contrary, we have recently developed a new relativistic mean-field (RMF) model [17]¹ that predicts a stiffer EOS of NS matter with the antikaon condensed phase than the EOS of normal NS matter without it. The model is based on the extended Zimanyi-Moszkowski (EZM) model [18-25] that has the field-dependent meson-baryon coupling constants in dense nuclear medium but the free meson-kaon coupling constants. It has been found that the abundance of antikaons severely suppresses hyperons. Because the abundance of hyperons generally leads to a rather soft EOS [26], we have inversely a rather stiff EOS due to antikaon condensations. Consequently, we can reproduce the recently observed massive NSs [27-30] being heavier than $1.6M_\odot$, which are not reproduced in the EZM model [25] consisting of only baryons as hadrons.

Because the result of Ref. [17] is remarkably different from the other models, it is worthwhile to apply the model to hot PNS. The purpose of the present work is to revisit the possibility of low-mass BHs. The formulation of the hot isentropic PNS matter consisting of baryons and leptons is the same as Ref. [6]. Then, we have only to add the kaon contributions. Here, we follow the formulation of Ref. [9] but take into account both the K^- (K^+) and \bar{K}^0 (K^0) in the isospin doublet. The kaons are treated [31] on the same footing as the baryons.

The thermodynamic potential per volume $\tilde{\Omega} \equiv \Omega/V$ at a given temperature T is

$$\begin{aligned}
 \tilde{\Omega} = & \frac{1}{2} m_\sigma^2 \langle \sigma \rangle^2 + \frac{1}{2} m_\delta^2 \langle \delta_3 \rangle^2 + \frac{1}{2} m_{\sigma^*}^2 \langle \sigma^* \rangle^2 - \frac{1}{2} m_\omega^2 \langle \omega_0 \rangle^2 - \frac{1}{2} m_\rho^2 \langle \rho_{03} \rangle^2 - \frac{1}{2} m_\phi^2 \langle \phi_0 \rangle^2 \\
 & - 2T \sum_{\substack{B=p,n,\Lambda,\Sigma^+, \\ \Sigma^0,\Sigma^-, \Xi^0, \Xi^-}} \int_0^\infty \frac{d^3\mathbf{k}}{(2\pi)^3} \left\{ \ln \left[1 + \exp \left(\frac{\nu_B - E_{kB}^*}{T} \right) \right] \right. \\
 & \qquad \qquad \qquad \left. + \ln \left[1 + \exp \left(\frac{-\nu_B - E_{kB}^*}{T} \right) \right] \right\} \\
 & - T \sum_{l=e,\mu,\nu_e,\nu_\mu} \gamma_l \int_0^\infty \frac{d^3\mathbf{k}}{(2\pi)^3} \left\{ \ln \left[1 + \exp \left(\frac{\mu_l - e_{kl}}{T} \right) \right] + \ln \left[1 + \exp \left(\frac{-\mu_l - e_{kl}}{T} \right) \right] \right\} \\
 & + \sum_{K=\bar{K}^0, K^-} \frac{1}{2} (f\theta_K)^2 (\alpha_K + 2\mu_K V_K - \mu_K^2) \\
 & + T \sum_{K=\bar{K}^0, K^-} \int_0^\infty \frac{d^3\mathbf{k}}{(2\pi)^3} \left\{ \ln \left[1 - \exp \left(\frac{\nu_K - \omega_K}{T} \right) \right] + \ln \left[1 - \exp \left(\frac{-\nu_K - \omega_K}{T} \right) \right] \right\}.
 \end{aligned} \tag{1}$$

¹As compared with Ref. [17], Ref. [15] did not take into account the strange mesons while Ref. [16] employed the field-dependent meson-kaon coupling constants. Moreover, Refs. [15] and [16] employed the different formulation and calculus of antikaon condensation from Ref. [17], although they did not have significant effects on the results.

The Boltzmann constant is set as a unit. The first to third lines in the right hand side are formally the same as Eq. (1) in Ref. [6]. The fourth line is the contribution of the s-wave antikaon condensations, in which f is the pion decay constant, θ_K is the condensate amplitude, V_K is the vector potential of kaon [17], μ_K is the chemical potential of kaon and α_K is defined by

$$\alpha_K = M_K^{*2} - V_K^2. \quad (2)$$

M_K^* is the effective mass of kaon [17] in strange baryonic matter. The fifth line in Eq. (1) is the thermal kaon contribution, in which

$$\omega_K = \sqrt{\mathbf{k}^2 + M_K^{*2}}, \quad (3)$$

$$\nu_K = \mu_K - V_K. \quad (4)$$

The nonlinear equations, which determine the three independent effective masses $M_p^* = m_p^* M_N$, $M_n^* = m_n^* M_N$ and $M_\Lambda^* = m_\Lambda^* M_\Lambda$ and the three independent vector potentials V_p , V_n and V_Λ , are

$$\begin{aligned} & C_{VN} \rho_p + \sum_{Y \neq \Lambda} (g_{nn\rho}^* g_{YY\omega}^* + g_{nn\omega}^* g_{YY\rho}^* I_{3Y} - C_{V\Lambda} g_{nn\rho}^* g_{YY\phi}^*) \rho_Y \\ & - \sum_K (g_{nn\rho}^* g_{KK\omega} - g_{nn\omega}^* g_{KK\rho} I_{3K} - C_{V\Lambda} g_{nn\rho}^* g_{KK\phi}) \rho_K \\ & - m_\omega^2 g_{nn\rho}^* \langle \omega_0 \rangle - m_\rho^2 g_{nn\omega}^* \langle \rho_{03} \rangle + C_{V\Lambda} m_\phi^2 g_{nn\rho}^* \langle \phi_0 \rangle = 0, \end{aligned} \quad (5)$$

$$\begin{aligned} & C_{VN} \rho_n + \sum_{Y \neq \Lambda} (g_{pp\rho}^* g_{YY\omega}^* - g_{pp\omega}^* g_{YY\rho}^* I_{3Y} - C_{V\Lambda} g_{pp\rho}^* g_{YY\phi}^*) \rho_Y \\ & - \sum_K (g_{pp\rho}^* g_{KK\omega} + g_{pp\omega}^* g_{KK\rho} I_{3K} - C_{V\Lambda} g_{pp\rho}^* g_{KK\phi}) \rho_K \\ & - m_\omega^2 g_{pp\rho}^* \langle \omega_0 \rangle + m_\rho^2 g_{pp\omega}^* \langle \rho_{03} \rangle + C_{V\Lambda} m_\phi^2 g_{pp\rho}^* \langle \phi_0 \rangle = 0, \end{aligned} \quad (6)$$

$$\sum_Y g_{YY\phi}^* \rho_Y - \sum_K g_{KK\phi} \rho_K - m_\phi^2 \langle \phi_0 \rangle = 0, \quad (7)$$

$$\begin{aligned} & M_N \rho_{Sp} + \sum_{Y \neq \Lambda} \frac{\partial m_Y^*}{\partial m_p^*} M_Y \rho_{SY} + \sum_{Y \neq \Lambda} \frac{\partial V_Y}{\partial m_p^*} \rho_Y + \sum_K \left(\frac{\partial M_K^*}{\partial m_p^*} \rho_{SK} + \frac{\partial V_K}{\partial m_p^*} \rho_K \right) \\ & + m_\sigma^2 \langle \sigma \rangle \frac{\partial \langle \sigma \rangle}{\partial m_p^*} + m_\delta^2 \langle \delta_3 \rangle \frac{\partial \langle \delta_3 \rangle}{\partial m_p^*} + m_{\sigma^*}^2 \langle \sigma^* \rangle \frac{\partial \langle \sigma^* \rangle}{\partial m_p^*} \\ & - m_\omega^2 \langle \omega_0 \rangle \frac{\partial \langle \omega_0 \rangle}{\partial m_p^*} - m_\rho^2 \langle \rho_{03} \rangle \frac{\partial \langle \rho_{03} \rangle}{\partial m_p^*} - m_\phi^2 \langle \phi_0 \rangle \frac{\partial \langle \phi_0 \rangle}{\partial m_p^*} = 0, \end{aligned} \quad (8)$$

$$\begin{aligned}
 & M_N \rho_{Sn} + \sum_{Y \neq \Lambda} \frac{\partial m_Y^*}{\partial m_n^*} M_Y \rho_{SY} + \sum_{Y \neq \Lambda} \frac{\partial V_Y}{\partial m_n^*} \rho_Y + M_N \sum_K \left(\frac{\partial M_K^*}{\partial m_n^*} \rho_{SK} + \frac{\partial V_K}{\partial m_n^*} \rho_K \right) \\
 & + m_\sigma^2 \langle \sigma \rangle \frac{\partial \langle \sigma \rangle}{\partial m_n^*} + m_\delta^2 \langle \delta_3 \rangle \frac{\partial \langle \delta_3 \rangle}{\partial m_n^*} + m_{\sigma^*}^2 \langle \sigma^* \rangle \frac{\partial \langle \sigma^* \rangle}{\partial m_n^*} \\
 & - m_\omega^2 \langle \omega_0 \rangle \frac{\partial \langle \omega_0 \rangle}{\partial m_n^*} - m_\rho^2 \langle \rho_{03} \rangle \frac{\partial \langle \rho_{03} \rangle}{\partial m_n^*} - m_\phi^2 \langle \phi_0 \rangle \frac{\partial \langle \phi_0 \rangle}{\partial m_n^*} = 0, \tag{9}
 \end{aligned}$$

$$\begin{aligned}
 & \sum_Y \frac{\partial m_Y^*}{\partial m_\Lambda^*} M_Y \rho_{SY} + \sum_{Y \neq \Lambda} \frac{\partial V_Y}{\partial m_\Lambda^*} \rho_Y + \sum_K \left(\frac{\partial M_K^*}{\partial m_\Lambda^*} \rho_{SK} + \frac{\partial V_K}{\partial m_\Lambda^*} \rho_K \right) \\
 & + m_{\sigma^*}^2 \langle \sigma^* \rangle \frac{\partial \langle \sigma^* \rangle}{\partial m_\Lambda^*} - m_\phi^2 \langle \phi_0 \rangle \frac{\partial \langle \phi_0 \rangle}{\partial m_\Lambda^*} = 0, \tag{10}
 \end{aligned}$$

where

$$C_{VN} = g_{pp\omega}^* g_{nn\rho}^* + g_{nm\omega}^* g_{pp\rho}^*, \tag{11}$$

$$C_{V\Lambda} = g_{\Lambda\Lambda\omega}^* / g_{\Lambda\Lambda\phi}^*. \tag{12}$$

The terms for baryons and mean-fields in Eqs. (5)-(10) are formally the same as Eqs. (42)-(47) in Ref. [6].² The isospin of kaon is defined by $I_{3K} = (1, -1)$ for $K = (\bar{K}^0, K^-)$. The terms for kaons include the densities defined by

$$\rho_K = (f \theta_K)^2 \nu_K + \int_0^\infty \frac{d^3 \mathbf{k}}{(2\pi)^3} [n_{kK}(T) - \bar{n}_{kK}(T)], \tag{13}$$

$$\rho_{SK} = (f \theta_K)^2 M_K^* + \int_0^\infty \frac{d^3 \mathbf{k}}{(2\pi)^3} \frac{M_K^*}{\omega_K} [n_{kK}(T) + \bar{n}_{kK}(T)], \tag{14}$$

where the Bose-Einstein distribution functions are

$$n_{kK}(T) = \left[\exp \left(\frac{\omega_K - \nu_K}{T} \right) - 1 \right]^{-1}, \tag{15}$$

$$\bar{n}_{kK}(T) = \left[\exp \left(\frac{\omega_K + \nu_K}{T} \right) - 1 \right]^{-1}. \tag{16}$$

²In Ref. [6] we mistyped Eqs. (45) and (46) so that their third terms include unnecessary factor M_Y .

The energy density is given by

$$\begin{aligned}
 \mathcal{E} &= \frac{1}{2} m_\sigma^2 \langle \sigma \rangle^2 + \frac{1}{2} m_\delta^2 \langle \delta_3 \rangle^2 + \frac{1}{2} m_{\sigma^*}^2 \langle \sigma^* \rangle^2 - \frac{1}{2} m_\omega^2 \langle \omega_0 \rangle^2 - \frac{1}{2} m_\rho^2 \langle \rho_{03} \rangle^2 - \frac{1}{2} m_\phi^2 \langle \phi_0 \rangle^2 \\
 &+ 2 \sum_{\substack{B=p,n,\Lambda,\Sigma^+, \\ \Sigma^0,\Sigma^-, \Xi^0,\Xi^-}} \left\{ \left(\int_0^\infty \frac{d^3 \mathbf{k}}{(2\pi)^3} E_{kB}^* [n_{kB}(T) + \bar{n}_{kB}(T)] \right) + V_B \rho_B \right\} \\
 &+ \sum_{l=e,\mu,\nu_e,\nu_\mu} \gamma_l \int_0^\infty \frac{d^3 \mathbf{k}}{(2\pi)^3} e_{kl} [n_{kl}(T) + \bar{n}_{kl}(T)] + \sum_{K=\bar{K}^0, K^-} \frac{1}{2} (f \theta_K)^2 (\alpha_K + \mu_K^2) \\
 &+ \sum_{K=\bar{K}^0, K^-} \int_0^\infty \frac{d^3 \mathbf{k}}{(2\pi)^3} [(\omega_K + V_K) n_{kK}(T) + (\omega_K - V_K) \bar{n}_{kK}(T)], \tag{17}
 \end{aligned}$$

where the first to third lines in the right hand side are formally the same as Eq. (55) in Ref. [6].

As mentioned first, our investigation is based on a static approach, which assumes that a newly born NS is in chemical-equilibrium state:

$$\mu_i = b_i \mu_n - q_i (\mu_e - \mu_{\nu_e}), \tag{18}$$

$$\mu_\mu - \mu_{\nu_\mu} = \mu_e - \mu_{\nu_e}, \tag{19}$$

where μ_i , b_i and q_i are the chemical potential, the baryon number and the charge of each particle. The properties of a newly born NS matter below the K^- condensation threshold $\mu_{K^-} < M_{K^-}^* + V_{K^-}$, is determined by solving Eqs. (5)-(10) with $\theta_{\bar{K}^0} = \theta_{K^-} = 0$ under the baryon number conservation

$$\rho_T = \sum_{\substack{B=p,n,\Lambda,\Sigma^+, \\ \Sigma^0,\Sigma^-, \Xi^0,\Xi^-}} \rho_B, \tag{20}$$

the charge neutral condition

$$\sum_{i=B,l,K} q_i \rho_i = 0, \tag{21}$$

the lepton number conservation [32]

$$Y_{Le} = \frac{\rho_e + \rho_{\nu_e}}{\rho_T} = 0.4, \tag{22}$$

$$Y_{L\mu} = \frac{\rho_\mu + \rho_{\nu_\mu}}{\rho_T} = 0, \tag{23}$$

and the isentropic condition

$$s = \frac{\mathcal{E} + P - (\mu_n + Y_{Le}\mu_{\nu_e}) \rho_T}{T\rho_T}, \quad (24)$$

by means of 11-dimensional Newton-Raphson method so that the effective masses M_p^* , M_n^* and M_Λ^* , the vector potentials V_p , V_n and V_Λ , the chemical potentials μ_n , μ_e , μ_{ν_e} and μ_{ν_μ} and the temperature T are determined selfconsistently. The pressure is given by $P = -\tilde{\Omega}$.

Above the K^- condensation threshold but below the \bar{K}^0 condensation threshold $\mu_{\bar{K}^0} < M_{\bar{K}^0}^* + V_{\bar{K}^0}$, Eqs. (5)-(10) with $\theta_{\bar{K}^0} = 0$ and Eqs. (20)-(24) are solved together with

$$M_{K^-}^* + V_{K^-} = \mu_{K^-}, \quad (25)$$

by means of 12-dimensional Newton-Raphson method so that we have the effective masses, the vector potentials, the chemical potentials, the temperature and the K^- condensation amplitude $f\theta_{K^-}$. Above the \bar{K}^0 condensation threshold, Eqs. (5)-(10) and (20)-(25) are solved together with

$$M_{\bar{K}^0}^* + V_{\bar{K}^0} = \mu_{\bar{K}^0}, \quad (26)$$

by means of 13-dimensional Newton-Raphson method so that we have the effective masses, the vector potentials, the chemical potentials, the temperature and the K^- and \bar{K}^0 condensation amplitudes.

In the present calculation, we use the same meson-baryon coupling constants as the EZM-P in Ref. [25] and the same meson-kaon coupling constants as Ref. [17] for the antikaon optical potential $U_{\bar{K}}(\rho_{NM}) = -170\text{MeV}$ in a saturated nuclear matter. Within the EZM model their values are the most reasonable ones at present. We assume [3,6] that the newly born NS is composed of three layers. The first is the neutrino-opaque inner core in the region $\rho_T \geq 0.1\text{fm}^{-3}$ with the entropy per baryon $s = 1$. The second is the neutrino-trapped hot shocked envelope in the region $6 \times 10^{-4}\text{fm}^{-3} \leq \rho_T \leq 0.02\text{fm}^{-3}$ with $s = 5$. The third is the neutrino-transparent cold outer crust in the region $\rho_T < 6 \times 10^{-4}\text{fm}^{-3}$. The EOSs of the first and second layers are calculated using our EZM model. Of course, the kaon-condensed phases appear only in the inner core. For the third layer we employ the EOSs of Feynman-Metropolis-Teller, Baym-Pethick-Sutherland and Negele-Vautherin from Ref. [33]. The EOSs of the transition regions between the first and second layers and between the second and third layers are obtained by linear interpolation.

Figure 1 shows the particle fractions in the inner core of newly born NS. Our calculation is stopped at $\rho_T = 1.37\text{fm}^{-3}$, above which the effective mass of neutron becomes negative. This is not a problem because the central baryon density $\rho_T = 0.944\text{fm}^{-3}$ of the most massive PNS is lower than the limit. There appear no Σ hyperons because

their fractions are lower than 10^{-4} . The threshold densities of K^- and \bar{K}^0 condensed phases are $\rho_T = 0.53\text{fm}^{-3}$ and $\rho_T = 0.74\text{fm}^{-3}$, respectively. They are clearly seen as the rapid decreases of Ξ hyperons, which are therefore poor in newly born NS. As compared with the cold deleptonized NS, the thermal K^- distribution in newly born NS shifts the threshold of K^- condensation to higher density because of the charge neutral condition. (See Fig. 4 in Ref. [17].) Above the threshold, the proton fraction becomes larger, and so the increase of Λ becomes slow because of the baryon number conservation. To the contrary, the thermal \bar{K}^0 distribution in newly born NS shifts the threshold of \bar{K}^0 condensation to lower density as compared with the cold NS. This is because the thermal distribution reduces the left hand side of Eq. (26). Above the threshold, the NS matter tends to recover the isospin symmetry. Consequently, the increase of K^- is restrained, and the proton turns to decrease but the neutron turns to increase. The isospin symmetry is recovered at $\rho_T = 0.70\text{fm}^{-3}$, above which the isovector-scalar mean-field changes its sign and so the symmetry is broken again.

The red and blue curves in Fig. 2 show the EOSs of the inner core in newly born NS and cold NS, respectively. The latter is the result of Ref. [17]. It is remarkable that both the EOSs are almost the same. In our EZM model [17] the K^- condensation leads to a stiff EOS of the cold NS. (The \bar{K}^0 condensation does not contribute directly to the EOS.) Therefore, the delayed K^- condensation threshold in Fig. 1 inversely softens the EOS of newly born NS. Consequently, the thermal contributions to the EOS are canceled out. The kaon condensations in the EOSs are the second-order phase transitions. This is an attractive feature of our EZM model as was discussed in Ref. [17]. Then, we calculate the properties of NSs by integrating the Tolman-Oppenheimer-Volkov equation [34]. Figure 3 shows the gravitational masses of newly born NS and cold NS as functions of the central energy density. Because of the EOSs in Fig. 2 the red and blue curves coincide above $\mathcal{E}_C = 6 \times 10^{14}\text{g/cm}^3$. The maximum mass is $M_G = 1.95M_\odot$ on both the curves, while the minimum mass of newly born NS is $M_G = 1.14M_\odot$. Figure 4 shows the relations between the gravitational mass and the radius. Because of the hot shocked envelope the radius $R = 14.27\text{km}$ of the newly born NS with maximum mass is larger than the radius $R = 11.64\text{km}$ of the most massive cold NS. The radius of the newly born NS with minimum mass is $R = 41.22\text{km}$.

Figure 5 shows the relations between the gravitational and baryonic mass of newly born NS and cold NS. The maximum baryonic mass $M_B = 2.15M_\odot$ on the red curve is lower than $M_B = 2.25M_\odot$ on the blue curve. The result indicates that the delayed collapse of a newly born NS to a low-mass BH cannot be expected against the suggestion [1] by Brown and Bethe. Is our prediction is reliable? Because our previous work without kaons [6] supports low-mass BHs, the present result is certainly due to the contributions of kaons. Moreover, the minimum mass of newly born NS plays a fundamental role in assessing the reliability, because it is sensitive [3,6,35-37] to our choice for the profile of hot shocked envelope. As shown by the lower vertical dashed line in Fig. 5, the

minimum baryonic mass $M_B = 1.154M_\odot$ of newly born NS determines the minimum gravitational mass of cold NS. The obtained value $M_G = 1.08M_\odot$ is consistent with the range $M_G = 1.35 \pm 0.27M_\odot$ [38] of the well observed NS masses except for the recently observed massive NSs [27-30]. This indicates that our calculation is reasonable.

For fairness we should mention a defect of our result. As shown by the upper vertical dashed line in Fig. 5, the maximum baryonic mass of newly born NS also determines the maximum gravitational mass of cold NS, which is lower than the maximum value $M_G = 1.95M_\odot$ on the blue curve. The obtained most massive NS with $M_G = 1.88M_\odot$ and $R = 12.54\text{km}$ lies just below the mass-radius relation [30,39] of EXO 0748-676. The defect is however solved by mass accretion of only $0.02M_\odot$. Consequently, we conclude that the delayed collapse due to kaon condensation is questionable, although there still remain uncertainties of the kaon interactions in dense strange hadronic matter.

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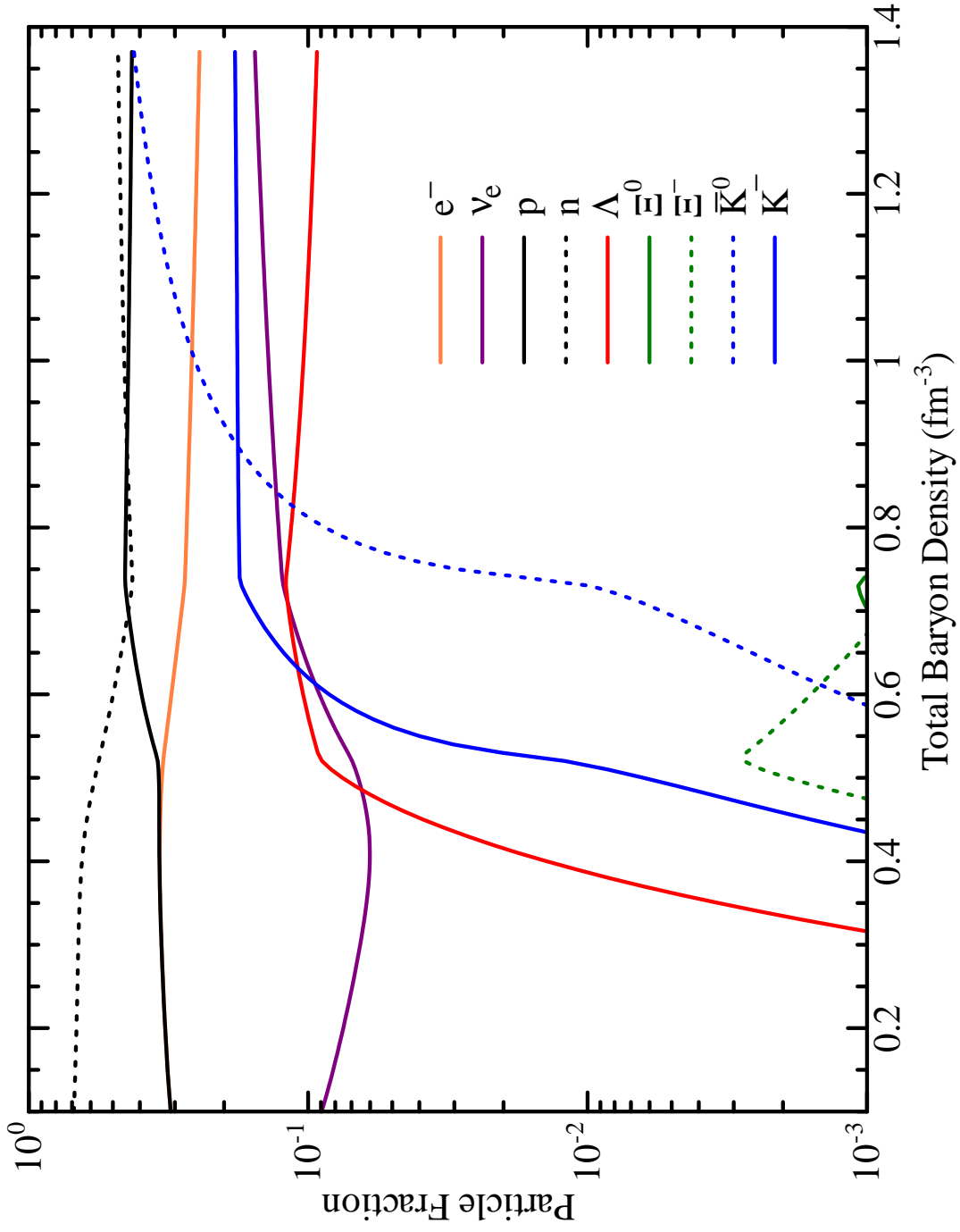


Figure 1: The particle fractions in the inner core of newly born NS as functions of total baryon density.

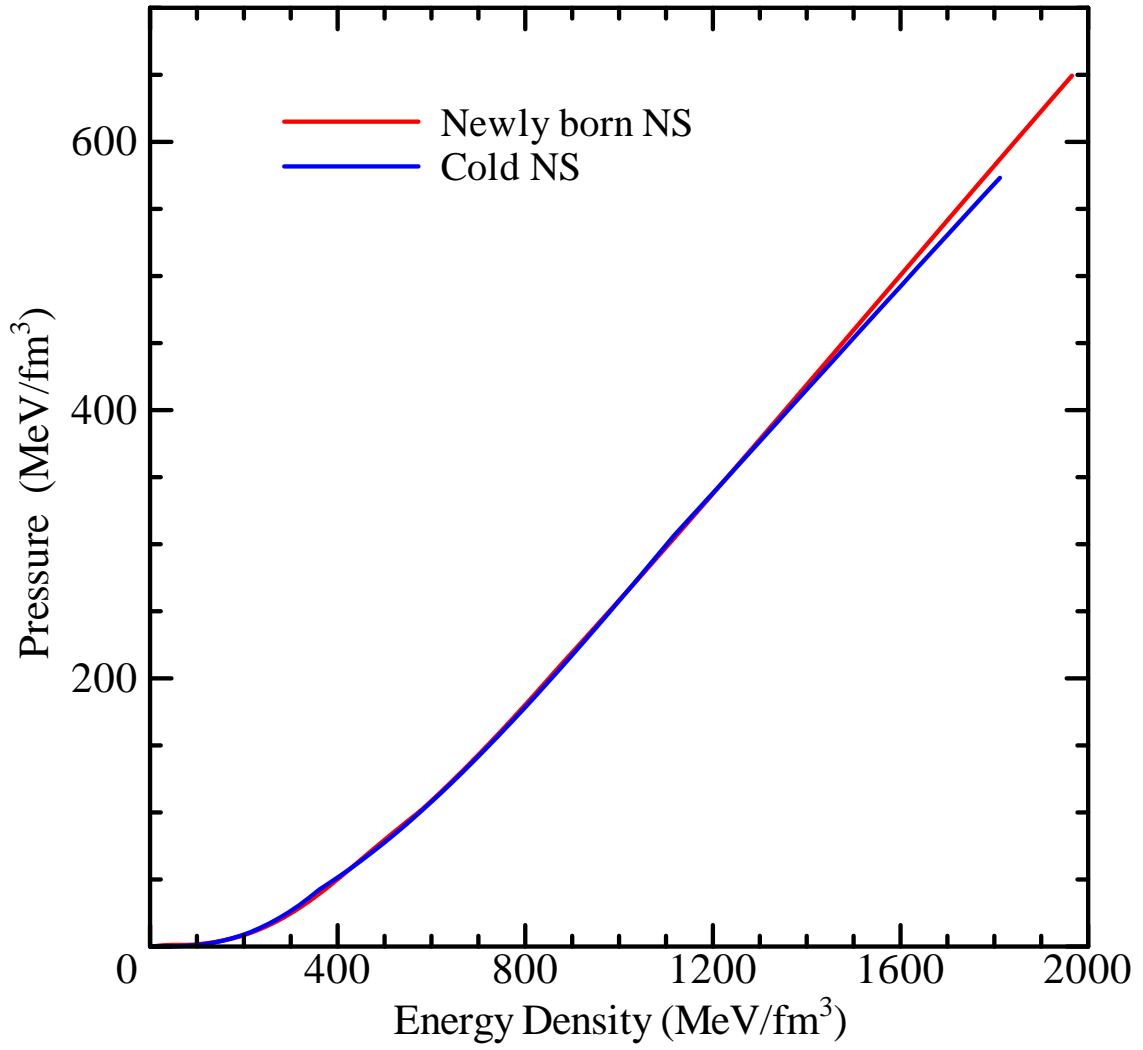


Figure 2: The EOSs of the inner core in newly born NS and cold NS.

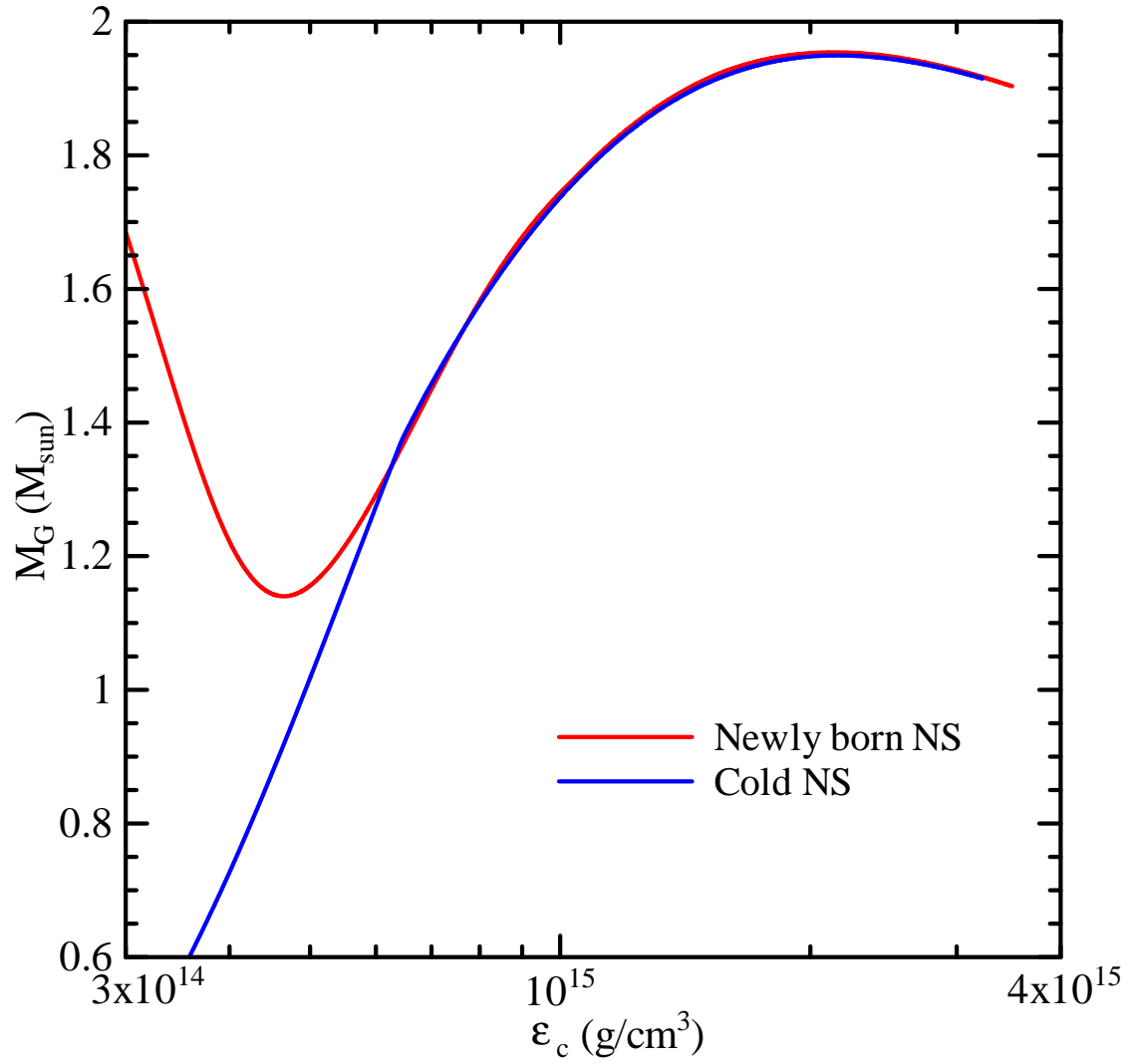


Figure 3: The gravitational masses of newly born NS and cold NS as functions of the central energy density.

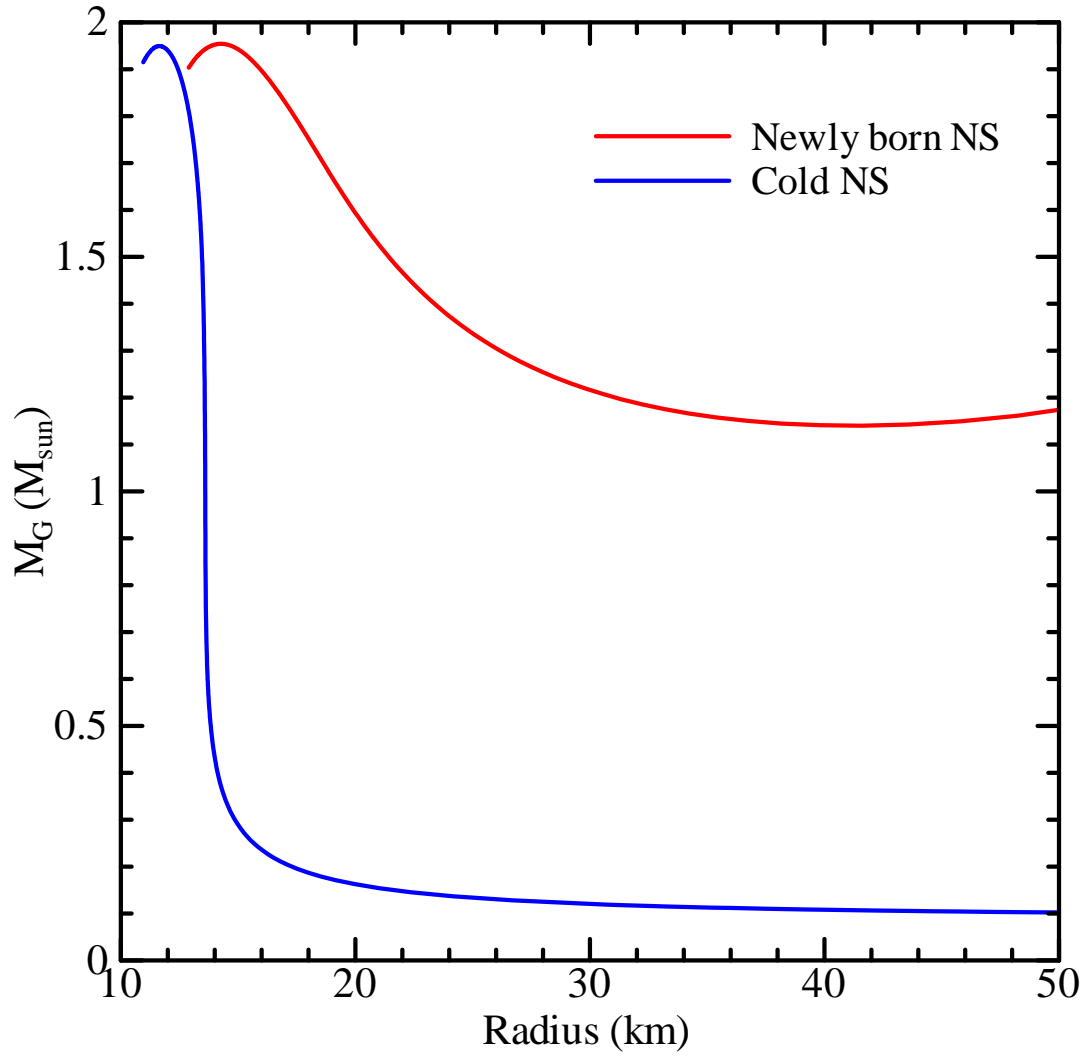


Figure 4: The relations between the gravitational mass and the radius of newly born NS and cold NS.

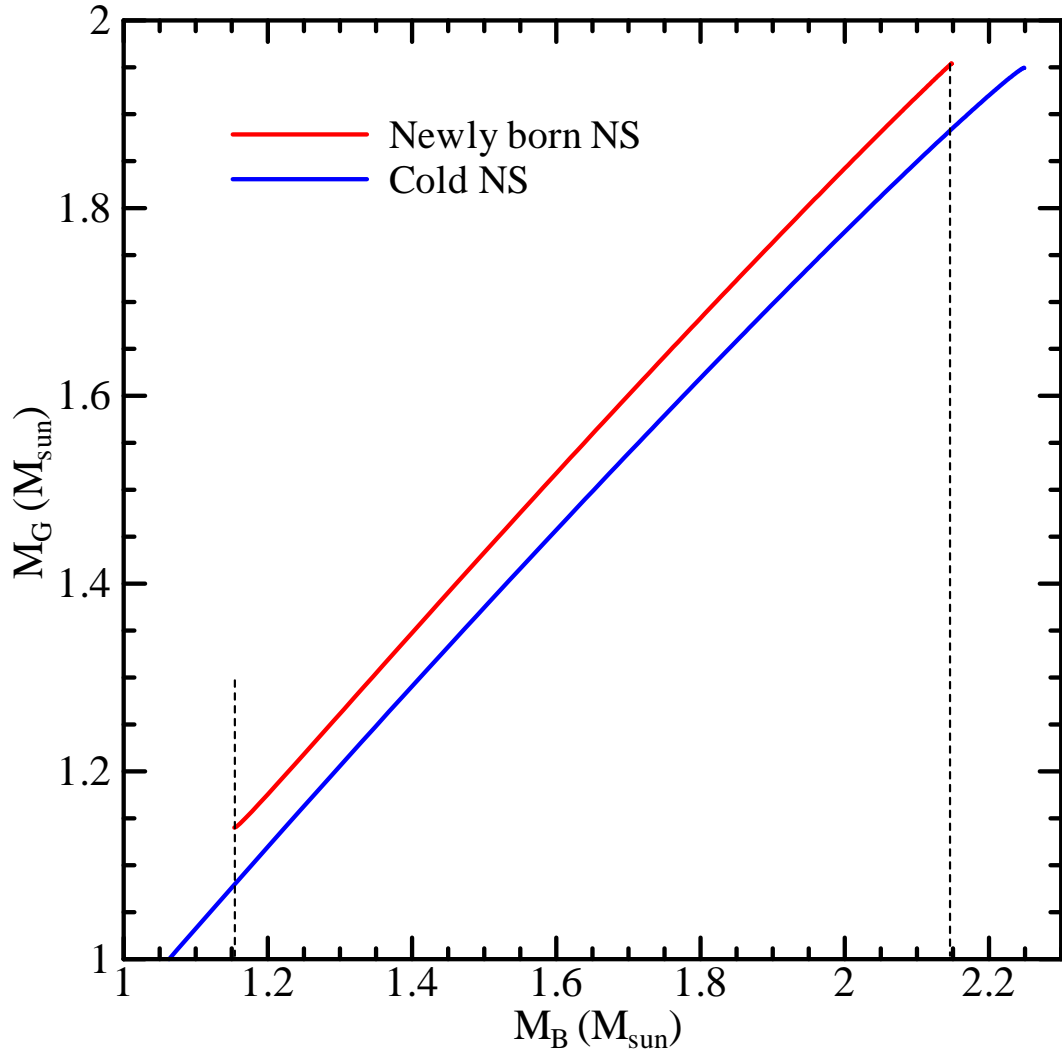


Figure 5: The relations between the gravitational and baryonic masses of newly born NS and cold NS. The vertical dashed lines indicate the baryonic masses corresponding the minimum and maximum gravitational masses of newly born NSs.